HPC/RPC Review Name KEY Period______
Properties of Exponentials and Logarithms
and Solving Exponential and Logarithmic equations

Write the equation in its equivalent exponential form.

1) $\log_4 16 = 2$

$4^2 = 16$

2) $\log_b 64 = 2$

$b^2 = 64$

Write the equation in its equivalent logarithmic form.

3) $6^3 = 216$

$\log_6 216 = 3$

4) $\sqrt[3]{343} = 7 \Rightarrow 343^{1/3} = 7$

$\log_{343} 7 = 1/3$

Solve the exponential equation. Use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

5) $10^x = 3.19$

$\log 3.19 = x$

$x = 0.504$

6) $e^{(3x)} = 5$

$\ln e^{3x} = \ln 5$

$3x = 1.609$

$x = 0.536$

7) $e^{(4x-5)} - 2 = 1182$

$\ln e^{4x-5} = \ln 1184$

$4x-5 = 7.077$

$4x = 12.077$

$x = 3.019$

Solve the logarithmic equation. Be sure to reject any value that is not in the domain of the original logarithmic expressions. Give the exact answer.

8) $\log_3 (x-5) + \log_3 (x-11) = 3$

$\log_3 (x-5)(x-11) = 3$

$3^3 = x^2 - 11x - 5x + 55$

$27 = x^2 - 16x + 55$

$x^2 - 16x + 28 = 0$

$(x-14)(x-2) = 0$

$\boxed{x=14}$ $\cancel{x=2}$

9) $4 + 8 \ln x = 8$

$8 \ln x = 4$

$e^{\ln x} = e^{1/2}$

$x = e^{1/2}$ $x = 1.649$

or

$8\ln x = 4$

$\ln x = \frac{1}{2}$

$\ln x = \boxed{\log_e x = \frac{1}{2}}$

$e^{1/2} = x$

$x = 1.649$

10) $\log_2 (x+3) = 4 + \log_2 (x-2)$

$\log_2(x+3) - \log_2(x-2) = 4$

$\log_2 \frac{x+3}{x-2} = 4$

$2^4 = \frac{x+3}{x-2}$

$16(x-2) = x+3$

$16x - 32 = x+3$

$15x = 35$

$x = 7/3$

11) $\log (3+x) - \log (x-2) = \log 2$

$\log \frac{3+x}{x-2} = \log 2 \rightarrow \frac{3+x}{x-2} = 2 \rightarrow 2x - 4 = 3 + x$

$x = 7$

12) $\ln x + \ln (x+1) = \ln 6$

$\ln (x^2 + x) = \ln 6$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$\boxed{x=2}$ $\cancel{x \neq -3}$

13) $\log (x+20) - \log 2 = \log (3x+4)$

$\log \frac{x+20}{2} = \log 3x + 4$

$\frac{x+20}{2} = \frac{3x+4}{1}$

$x + 20 = 6x + 8$

$12 = 5x$

$x = 12/5$

Properties of Exponentials and Logarithms and Solving Exponential and Logarithmic equations

Solve the problem.

1) The population of wolves in a state park after t years is modeled by the function $P(t) = \frac{800}{1 + 99e^{-0.3t}}$.

What was the initial population of wolves?

$P(0) = \frac{800}{1+99} = 8$

What is the maximum sustainable population?

800

About how many years will it take for the wolf population to reach 200?

$\frac{200}{1} = \frac{800}{1+99e^{-0.3t}}$

$800 = 200(1+99e^{-0.3t})$

$4 = 1+99e^{-0.3t}$

$3 = 99e^{-0.3t}$

$\frac{3}{99} = e^{-0.3t}$

$\ln\left(\frac{1}{33}\right) = -0.3t$

$-3.4965 = -0.3t$

$\boxed{11.6 \text{ years} = t}$

2) How long will it take for \$5500 to grow to \$22,600 at an interest rate of 3% if the interest is compounded continuously? Round the number of years to the nearest hundredth.

$22{,}600 = 5500e^{.03t}$

$4.109 = e^{.03t}$

$\ln(4.109) = .03t$

$1.4132 = .03t$

$47.11 = t$

3) Find how long it will take for \$6200 invested at 9.275% per year compounded daily to triple in value. Find the answer to the nearest year.

$18600 = 6200\left(1 + \frac{.09275}{365}\right)^{365t}$

$3 = 1.00025^{365t}$

$\log_{1.00025} 3 = 365t$

$4323.93 = 365t$

$11.85 = t$

About 12 years

Use Newton's Law of Cooling, $T = C + (T_0 - C)e^{-kt}$, for problems 4 & 5.

4) A lasagna removed from the oven has a temperature of 430°F. It is left sitting in a room that has a temperature of 65°F. After 6 minutes, the temperature of the lasagna is 320°F. Use Newton's Law of Cooling to find a model for the temperature of the lasagna, T, after t minutes.

$$320 = 65 + (430 - 65)e^{-k(6)}$$
$$320 = 65 + 365e^{-6k}$$
$$255 = 365e^{-6k}$$
$$0.6986 = e^{-6k}$$
$$\ln(0.6986) = -6k$$
$$-0.3586 = -6k$$
$$0.0598 = k$$

$$T = 65 + (430 - 65)e^{-0.0598t}$$
$$T = 65 + 365e^{-0.0598t}$$

5) A cake is removed from an oven at 325 °F and cools to 150 °F after 25 minutes in a room 68 °F. How long will it take the cake to cool to 113 °F?

$$150 = 68 + (325 - 68)e^{-k(25)}$$
$$82 = 257e^{-25k}$$
$$0.319 = e^{-25k}$$
$$\ln 0.319 = -25k$$
$$-1.1424 = -25k$$
$$0.0457 = k$$

$$113 = 68 + (257)e^{-0.0457t}$$
$$45 = 257e^{-0.0457t}$$
$$0.1751 = e^{-0.0457t}$$
$$\ln(0.1751) = -0.0457t$$
$$-1.742 = -0.0457t$$
$$38.1 \text{ minutes} = t$$