HPC/RPC Review
Composition of Functions

Name Key

Date________________**Period**_____

PC Reporting Strand: Functions (Compose and Transform Functions) Score______________

1. Given the functions: $f(x) = 4x^2 + 2$ and $g(x) = x - 2$:

Identify the domain of $f(x)$ and $g(x)$.

$f(x)$: D: $(-\infty, \infty)$

$g(x)$: D: $(-\infty, \infty)$

Evaluate $f(g(1))$, $g(f(-3))$ and $g(g(5))$.

$g(1) = -1 \quad f(g(1)) = 6$

$f(-3) = 38 \quad g(f(-3)) = 36$

$g(5) = 3 \quad g(g(5)) = 1$

Evaluate $f(g(x))$ and $g(f(x))$ and identify the domain of each.

$$\begin{aligned} f(g(x)) &= 4(x-2)(x-2) + 2 \\ &= 4[x^2 - 4x + 4] + 2 \\ &= 4x^2 - 16x + 16 + 2 \\ &= 4x^2 - 16x + 18 \end{aligned}$$

$$\begin{aligned} g(f(x)) &= 4x^2 + 2 - 2 \\ &= 4x^2 \end{aligned}$$

Evaluate $f(2g(x))$

$$2(g(x)) = 2(x-2) = 2x - 4$$

$$\begin{aligned} f(2g(x)) &= 4[(2x-4)(2x-4)] + 2 \\ &= 4[4x^2 - 16x + 16] + 2 \\ &= 16x^2 - 64x + 64 + 2 \\ &= 16x^2 - 64x + 66 \end{aligned}$$

A herbal supplement company makes an energy powder. The cost to produce the powder depends on the cost of ginkgo biloba. The cost of ginkgo biloba is dependent upon the size of the ginkgo biloba seed. The size of the seed is dependent upon the amount of lime in the soil.

The cost to manufacture the cookie is ***e = f(b)***

b Size seed in millimeters	12	15	18	21	24	27
e Cost of powder in dollars	0.04	.08	.12	0.16	.20	.24

The size of the pit is a function of temperature ***b= g(l).***

l lime content %	30	26	23	20	16	10
b Size seed in millimeters	27	24	21	18	15	12

Use the tables to evaluate

a. f(g(26)) = 0.20

b f(g(10)) = 0.04

2. Determine two functions, *f*(*x*) and *g*(*x*) that could be composed to create the given function.

a. $h(x) = \sqrt{3x - 7}$

$g(x) = 3x$
$f(x) = \sqrt{x-7}$

$g(x) = 3x-7$
$f(x) = \sqrt{x}$

$g(x) = \sqrt{3x-7}$
$f(x) = x$

b. $h(x) = \frac{4x}{(x-7)^3}$

$g(x) = \frac{4x}{(x-7)^3}$

$f(x) = x$

3. Analysis of a decomposition.

A student was given the following three compositions h(x) and then found the two functions f(x) and g(x) for each of the respective compositions. Determine if the student's solutions are correct. Please justify your answer algebraically followed with a written explanation.

a. $h(x) = \frac{3x^2+3}{\sqrt{2x^2+1}}$ $g(x) = x^2 + 1$ $f(x) = \frac{3}{\sqrt{2x}}$

Does $f(g(x)) = h(x)$

$$\frac{3}{\sqrt{2(x^2+1)}} = \frac{3}{\sqrt{2x^2+2}} \neq h(x)$$

NO

b. $h(x) = (x+1)\ ^2 - 3(x+1) + 4$ $g(x) = x + 1$ $f(x) = x^2 - 3x + 4$

$f(g(x)) = (x+1)^2 - 3(x+1) + 4 = h(x)$ ✓

yes

c. c. $h(x) = \sqrt[3]{x^2 - 9}$ $g(x) = x - 3$ $f(x) = \sqrt[3]{x^2}$

$f(g(x)) = \sqrt[3]{(x-3)^2} = \sqrt[3]{x^2 - 6x + 9}$

$f(g(x)) \neq h(x)$

NO

4.Find the inverse of the function and the domain of the inverse.

$f(x) = x^5 - 2$

Inverse: $f^{-1}(x) = \sqrt[5]{x+2}$

Domain: $(-\infty, \infty)$

① $y = x^5 - 2$ ③ $y^5 = x + 2$

② $x = y^5 - 2$ ④ $f^{-1}(x) = \sqrt[5]{x+2}$

$f(x) = \sqrt{4 - x^2}$

Inverse: $f^{-1}(x) = \sqrt{4-x^2}$

Domain: $[-2, 2]$

① $y = \sqrt{4-x^2}$

② $x = \sqrt{4-y^2}$

③ $x^2 = 4 - y^2$

$y^2 = 4 - x^2$

$y = \sqrt{4-x^2}$

$4 - x^2 \geq 0$

$-x^2 \geq -4$

$x^2 \leq 4$

$f(x) = \frac{6x+4}{4x+5}$

Inverse: $f^{-1}(x) = \frac{4-5x}{4x-6}$

Domain: $(-\infty, 1.5] \cup [1.5, \infty)$

① $y = \frac{6x+4}{4x+5}$

$\frac{x}{1} = \frac{6y+4}{4y+5}$

$x(4y+5) = 6y+4$

$4xy + 5x = 6y + 4$

$4xy - 6y = 4 - 5x$

$y(4x-6) = 4 - 5x$

$y = \frac{4-5x}{4x-6}$

$x \neq 3/2$

$f(x) = \sqrt[5]{\frac{2x}{5}} - 1$

Inverse: $f^{-1} = \frac{5}{2}(x+1)^5$

Domain: $(-\infty, \infty)$

① $y = \sqrt[5]{\frac{2x}{5}} - 1$

$x = \sqrt[5]{\frac{2y}{5}} - 1$

$x + 1 = \sqrt[5]{\frac{2y}{5}}$

$(x+1)^5 = \frac{2y}{5}$

$y = \frac{5}{2}(x+1)^5$

5. Determine if the given functions are inverses of each other.

a). $f(x) = -\frac{1}{x+1} - 3$ NO

$g(x) = -\frac{4}{x} + 2$

① $y = -\frac{4}{x} + 2$

② $x = -\frac{4}{y} + 2$

③ $x - 2 = \frac{4}{y}$

$y(x-2) = 4$

$xy - 2y = 4$

$y(x-2) = 4$

$y = \frac{4}{x-2}$

b) $f(x) = \frac{6+x}{2}$ yes

$h(x) = 2x - 6$

$f(h(x)) = \frac{6+2x-6}{2} = x$

c). $g(x) = \frac{1}{x-1} - 2$ NO comp

$f(x) = \frac{1}{x+2} + 1$ yes

$g(f(x)) = \frac{1}{\frac{1}{x+2} + \cancel{1} - 1} - 2$

$= \frac{1}{\left(\frac{1}{x+2}\right)} - 2$

$= \left(\frac{x+2}{1}\right) \cdot \frac{1}{1} - 2$

$= x + 2 - 2$

$= x$

d) $f(x) = \sqrt[3]{x-2}$ NO

$g(x) = -2x^3 + 3$

① $y = -2x^3 + 3$

$x = -2y^3 + 3$

$x - 3 = -2y^3$

$y^3 = \frac{x-3}{-2}$

$y = \sqrt[3]{\frac{x-3}{2}}$

6. Use the graphs and tables provided to find the values.

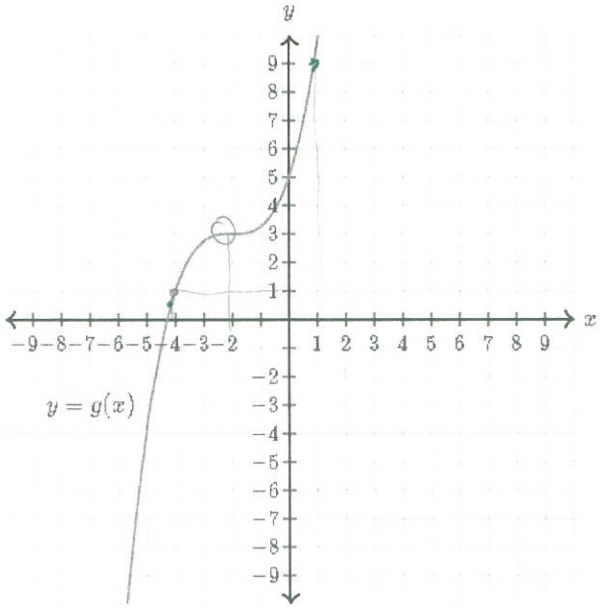
a)

x	9	−11	7	11	−6	−13
$f(x)$	11	4	−6	−1	−15	−5

$f^{-1}(-15)$ −6 $f^{-1}(f(93))$ 93 $f(f(7))$ −15 $f^{-1}(4) + f(9)$ 0

−11 +11

b)



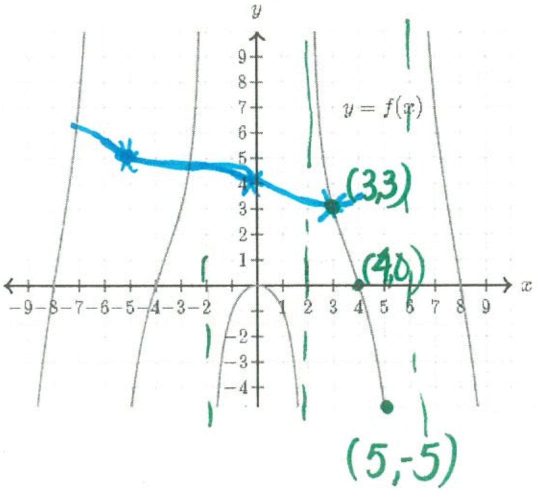
$g^{-1}(1)$ −4

$g^{-1}(g(-2))$ −2

$g(g(-4))$ 9

7. Determine if the following functions are invertible. If not, identify a domain that would produce an invertible function. Also, graph the inverse based on your domain.

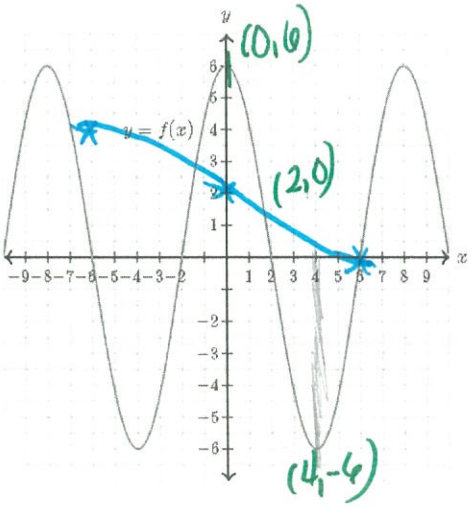
a)



Invertible? NO

Restricted Domain?

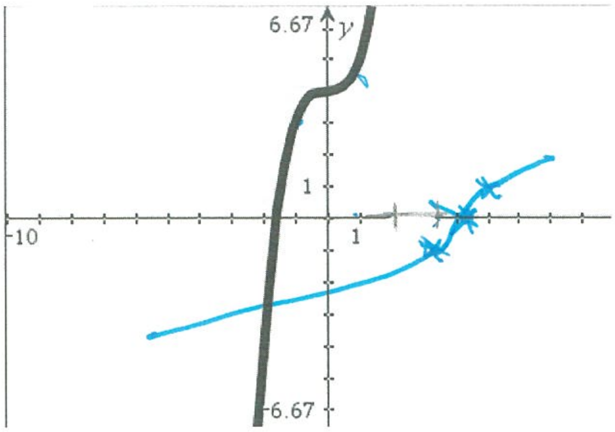
b)



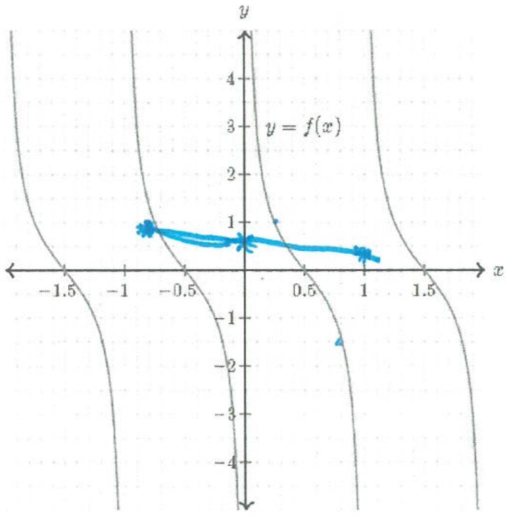
Invertible? NO

Restricted Domain? [0, 4]

c)



d)



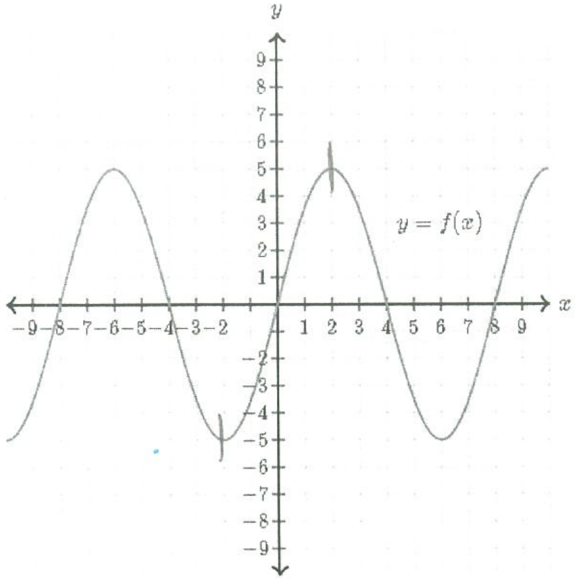
Invertible? yes

Restricted Domain? ____________________

Invertible? NO

Restricted Domain? [(0,1)

8. A student was given the following graph. The student gave the response below. Determine if the student's solutions are correct. Please justify your answer with a written explanation.



Student Response:

This is a non-invertible function because it fails the HLT. In order to turn this into an invertible function, I would restrict the domain as follows:

[0, 4]

[-2, 2]