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Precalculus
Solving Trigonometric Equations
Review

1. **Find the exact value of each expression.**

a. $\sin 315°$ $= \sin(270+45) = \sin 270\cos 45 + \cos 270 \sin 45$
$= (-1)\left(\frac{\sqrt{2}}{2}\right) + (0)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{2}$

b. $\tan 15°$ $= \tan(45-30) = \frac{\tan 45 - \tan 30}{1+\tan 45 \tan 30} = \frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}$ or

$\tan(60-45) = \frac{\tan 60 - \tan 45}{1+\tan 60 \tan 45}$ $\frac{\sqrt{3}-1}{1+\sqrt{3}}$

c. $\cos\frac{5\pi}{12}$ $= \cos\left(\frac{3\pi}{4}-\frac{\pi}{3}\right) = \cos\frac{3\pi}{4}\cdot\cos\frac{\pi}{3} + \sin\frac{3\pi}{4}\cdot\sin\frac{\pi}{3}$
$= -\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$
$= \frac{\sqrt{6}-\sqrt{2}}{4}$

2. **Write the expression as the tangent of an angle, then find the exact value.**

a. $\frac{\tan 155 + \tan 85}{1-\tan 155 \tan 85}$ $= \tan(155+85) = \tan 240 = \sqrt{3}$

b. $\sin\frac{15\pi}{36}\cos\frac{\pi}{6} - \cos\frac{15\pi}{36}\sin\frac{\pi}{6}$ $= \sin\left(\frac{15\pi}{36}-\frac{\pi}{6}\right) = \sin\left(\frac{15\pi}{36}-\frac{6\pi}{36}\right) =$
$\sin\left(\frac{9\pi}{36}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

c. $\cos\frac{2\pi}{3}\cos\frac{\pi}{6} + \sin\frac{2\pi}{3}\sin\frac{\pi}{6}$ $= \cos\left(\frac{2\pi}{3}-\frac{\pi}{6}\right) = \cos\left(\frac{4\pi}{6}-\frac{\pi}{6}\right) = \cos\frac{\pi}{2}$

$\downarrow$

0

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3. **Solve the trigonometric equation for all values of x over *the interval* [0, 2π]**

a. $3\tan x + 6 = 0$

$\tan x = -2$

Not possible

No Sol'n

b. $5\cot\theta = \sqrt{3} + 4\cot\theta$

$5\cot\theta - 4\cot\theta - \sqrt{3} = 0$

$\cot\theta - \sqrt{3} = 0$

$\cot\theta = \frac{\sqrt{3}}{1}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ or $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$

c. $12\sin^2 x - 3 = 6\sin^2 x$

$12\sin^2 x - 6\sin^2 x - 3 = 0$

$6\sin^2 x - 3 = 0$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

d. $4\cos^2 x - 3 = 0$

$4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\left(\pm\frac{\sqrt{3}}{2}, \pm\frac{1}{2}\right)$

e. $4\tan^2 x - 1 = \tan^2 x$

$4\tan^2 x - \tan^2 x - 1 = 0$

$3\tan^2 x - 1 = 0$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

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4. Solve the trigonometric equation for ***all values of x.***

a. $2\tan^2\theta \sin\theta = \frac{2}{\cot^2 x}$

$2\tan^2\theta\sin\theta = 2\tan^2\theta$
$2\tan^2\theta\sin\theta - 2\tan^2\theta = 0$
$2\tan^2\theta(\sin\theta - 1) = 0$
$2\tan^2\theta = 0$ $\quad$ $\sin\theta - 1 = 0$
$\tan^2\theta = 0$ $\quad$ $\sin\theta = 1$
$\tan\theta = 0$ $\quad$ $\theta = \frac{\pi}{2}$
$\theta = 0, \pi$
$\theta = \left\{0, \frac{\pi}{2}, \pi\right\} + 2n\pi$

b. $(\sec^2\theta - 1) = 2\tan\theta - 1$

$\tan^2\theta - 2\tan\theta + 1 = 0$
$(\tan\theta - 1)(\tan\theta - 1) = 0$
$\tan\theta = 1$
$\theta = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\} + 2n\pi$

c. $6\cos 2\theta - 1 = 4\cos 2\theta$

$6\cos 2\theta - 4\cos 2\theta - 1 = 0$
$2\cos 2\theta - 1 = 0$
$\cos 2\theta = \frac{1}{2}$
$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}$
$\theta = \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\} + 2n\pi$

d. $\csc^2 x - \cot x \csc^2 x = 0$

$\csc^2 x(1 - \cot x) = 0$
$\csc^2 x = 0$ $\quad$ $1 - \cot x = 0$
$\cot x = 1$
$\csc x = 0$ $\quad$ $\tan x = 1$
$\sin x = \frac{1}{0}$ ← undefined, $\quad$ $x = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\} + 2n\pi$

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e. $\cot^2 x - \sec x \cot^2 x = 0$

$\cot^2 x\,(1-\sec x) = 0$

$\cot^2 x = 0 \qquad 1-\sec x = 0$

$\cot x = \frac{0}{\pm 1} \qquad \sec x = 1$

$\tan x = \frac{\pm 1}{0} \qquad \cos x = 1$

$x = \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 0\right\} + 2n\pi$

f. $1 - \cos^2\theta = \sin\theta + 2$

$(1-\cos^2\theta) - \sin\theta - 2 = 0$

$\sin^2\theta - \sin\theta - 2 = 0$

$(\sin\theta - 2)(\sin\theta + 1) = 0$

$\sin\theta - 2 = 0 \qquad \sin\theta + 1 = 0$

$\sin\theta = 2 \qquad \sin\theta = -1$

$\downarrow$

NO SOLN $\qquad \theta = \left\{\frac{3\pi}{2}\right\} + 2n\pi$

g. $2\sin^4 x - 3\sin^2 x = -1$

$2\sin^4 x - 3\sin^2 x + 1 = 0$

$(2\sin^2 x - 1)(\sin^2 x - 1)$

$2\sin^2 x - 1 = 0 \qquad \sin^2 x - 1 = 0$

$\sin^2 x = \frac{1}{2} \qquad \sin^2 x = 1$

$\sin x = \pm\frac{1}{\sqrt{2}} \qquad \sin x = \pm 1$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$x = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}\right\} + 2n\pi$