

Honors PreCalculus Algebra Review

1. Expand the expression $2(x^2 - x)$

$$2x^2 - 2x$$

2. Factor $2x^3 + 4x^2$

$$2x^2(x+2)$$

$x = 0$ and -2

$$2x^2 = 0 \quad x+2 = 0$$

Simplify the following expressions. Assume that denominators are not zero.

3. $\frac{(uv^2)^3}{v^2u^3} \cdot \frac{u^3v^6}{u^3v^2} = v^4$

4. $(3x^2y^3)^{-2} = \frac{1}{9x^4y^6}$

In problems 5 and 6, find a) the distance between the two points, and b) the midpoint of the segment determined by the points.

5. $(-5, 0)$ and $(14, 0)$ $M = \left(\frac{-5+14}{2}, \frac{0+0}{2} \right) = (4.5, 0)$

$$d = \sqrt{(-5-14)^2 + (0-0)^2} = \sqrt{(-19)^2 + (0)^2} = \sqrt{361} = 19$$

6. $(-4, 3)$ and $(5, -1)$ $M = \left(\frac{-4+5}{2}, \frac{3-1}{2} \right) = (0.5, 1)$

$$d = \sqrt{(-4-5)^2 + (3+1)^2} = \sqrt{(-9)^2 + (4)^2} = \sqrt{81+16} = \sqrt{97}$$

In problems 7 and 8, show that the figure determined by the points is the indicated type.

7. Right triangle: $(-2, 1), (3, 11), (7, 9)$

$$XY = \sqrt{(-2-3)^2 + (1-11)^2} = \sqrt{125}$$

$$YZ = \sqrt{(3-7)^2 + (11-9)^2} = \sqrt{20}$$

$$ZX = \sqrt{(7+2)^2 + (9-1)^2} = \sqrt{145}$$

yes right triangle

8. Equilateral triangle: $(0, 1), (4, 1), (2, 1-2\sqrt{3})$ Yes

$$JK = \sqrt{(0-4)^2 + (1-1)^2} = \sqrt{16} = 4$$

$$KL = \sqrt{(4-2)^2 + (1-1+2\sqrt{3})^2} = \sqrt{(2)^2 + (2\sqrt{3})^2} = \sqrt{4+12} = 4$$

$$LJ = \sqrt{(2-0)^2 + (1-2\sqrt{3}-1)^2} = \sqrt{4+12} = \sqrt{16} = 4$$

In problems 9 and 10, find the standard form equation for the circle.

9. Center $(0, 0)$, radius 2

$$x^2 + y^2 = 4$$

10. Center $(5, -3)$, radius 4

$$(x-5)^2 + (y+3)^2 = 16$$

For problems 11 and 12, find the center and the radius of the circle.

11. $(x+5)^2 + (y+4)^2 = 9$

$(h, k) = (-5, -4)$
 $r = 3$

12. $x^2 + y^2 = 1$

$(h, k) = (0, 0)$
 $r = 1$

13. For a triangle with the following vertices,

a) find the length of each side, and b) show that it is a right triangle.

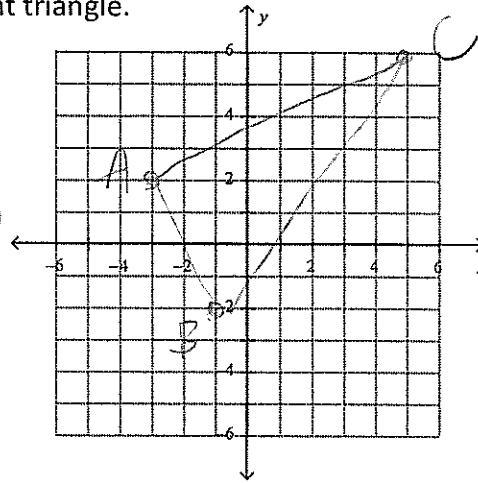
Vertices: $A(-3, 2), B(-1, -2), C(5, 6)$

$AB = \sqrt{(-3+1)^2 + (2+2)^2} = \sqrt{4+16} = \sqrt{20}$

$BC = \sqrt{(-1-5)^2 + (-2-6)^2} = \sqrt{36+64} = \sqrt{100} = 10$

$CA = \sqrt{(5+3)^2 + (6-2)^2} = \sqrt{64+16} = \sqrt{80}$

$a^2 + b^2 = c^2$
 $(\sqrt{20})^2 + (\sqrt{80})^2 = 10^2$
 $20 + 80 = 100$
 $100 = 100 \checkmark$



14. Let $(3, 5)$ be the midpoint of the line segment with endpoints $(-1, 1)$ and (a, b) . Determine a and b .

$3 = \frac{-1+a}{2}$ $5 = \frac{1+b}{2}$
 $6 = -1+a$ $10 = 1+b$
 $a = 7$ $b = 9$
 $\therefore (7, 9)$

15. Find the slope of the line through the points $(-1, -2)$ and $(4, -5)$.

$m = \frac{-5+2}{4+1} = -\frac{3}{5}$

16. Find an equation in point-slope form for the line through the point $(2, -5)$ with slope $m = -\frac{2}{3}$.

$-5 = -\frac{2}{3}(2) + b$ $b = -\frac{11}{3}$
 $-5 = \frac{4}{3} + b$ $y = -\frac{2}{3}x - \frac{11}{3}$
 $-\frac{15}{3} = \frac{4}{3} + b$

17. Find an equation of the line through the points $(-5, 4)$ and $(2, -5)$ in the general form $Ax + By + C = 0$

$\Rightarrow m = -\frac{9}{7}$
 $-5 = -\frac{9}{7}\left(\frac{2}{1}\right) + b$
 $-5 = -\frac{18}{7} + b$
 $\frac{7}{7}(-5) = -\frac{18}{7} + b$
 $-\frac{35}{7} = -\frac{18}{7} + b$
 $-\frac{17}{7} = b$
 $y = -\frac{9}{7}x - \frac{17}{7}$
 $\frac{9}{7}x + y + \frac{17}{7} = 0$
 $9x + 7y + 17 = 0$

For problems 18 – 23, find an equation in slope-intercept form for the line.

18. The line through (3, -2) with slope $m = \frac{4}{5}$

$$-2 = \frac{4}{5}(3) + b$$

$$-2 = \frac{12}{5} + b$$

$$b = -\frac{22}{5}$$

$$y = \frac{4}{5}x - \frac{22}{5}$$

19. The line through the points (-1, -4) and (3, 2).

$$m = \frac{2 - (-4)}{3 - (-1)} = \frac{6}{4} = \frac{3}{2}$$

$$b = -\frac{5}{2}$$

$$-4 = \frac{3}{2}(-1) + b$$

$$-4 = \frac{3}{2} + b$$

$$b = -\frac{11}{2}$$

$$y = \frac{3}{2}x - \frac{11}{2}$$

20. The line through (-2, 4) with slope $m = 0$

$$y = 4$$

21. The line $3x - 4y = 7$.

$$y = \frac{3}{4}x - \frac{7}{4}$$

22. The line through (2, -3) parallel to the line $2x + 5y = 3$.

$$y = -\frac{2}{5}x + \frac{3}{5}$$

$$-3 = -\frac{2}{5}(2) + b$$

$$b = -\frac{11}{5}$$

$$-\frac{15}{5} = -\frac{4}{5} + b$$

$$y = -\frac{2}{5}x - \frac{11}{5}$$

23. The line through (2, -3) perpendicular to the line $2x + 5y = 3$.

$$y = -\frac{5}{2}x + \frac{3}{2}$$

$$m = \frac{5}{2}$$

$$-3 = \frac{5}{2}(2) + b$$

$$b = -8$$

$$y = \frac{5}{2}x - 8$$

24. Consider the point (-6, 3) and line $L: 4x - 3y = 5$. Write an equation **a)** for the line passing through this point and parallel to L , and **b)** for the line passing through this point and perpendicular to L .

$$4x - 3y = 5$$

$$3y = 4x - 5$$

$$y = \frac{4}{3}x - \frac{5}{3}$$

$$m = \frac{4}{3}$$

// line

$$3 = \frac{4}{3}(-6) + b$$

$$3 = -8 + b$$

$$b = 11$$

$$y = \frac{4}{3}x + 11$$

⊥ line

$$3 = -\frac{3}{4}(-6) + b$$

$$3 = \frac{18}{4} + b$$

$$3 = \frac{9}{2} + b$$

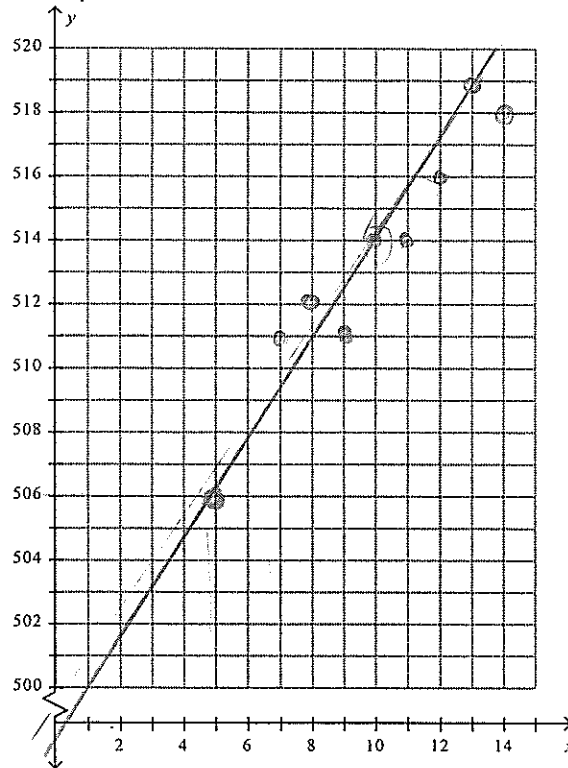
$$-\frac{3}{2} = b$$

$$y = -\frac{3}{4}x - \frac{3}{2}$$

25. The SAT scores are measured on an 800-point scale. The data in the following table shows the average SAT math score for several years.

Year	SAT Math Score
1995	506
1997	511
1998	512
1999	511
2000	514
2001	514
2002	516
2003	519
2004	518

- a) Let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth. Make a scatter plot of the data.



- b) Use the 1995 and 2000 data to write a linear equation for the average SAT math score y in terms of the year x . Superimpose the graph of the linear equation on the scatter plot in (a).

$$m = \frac{514 - 506}{10 - 5} = \frac{8}{5} = 1.6$$

$$\begin{aligned} 506 &= 1.6(5) + b \\ 506 &= 8 + b \\ 498 &= b \end{aligned}$$

$$y = 1.6x + 498$$

- c) Use the equation in (b) to estimate the average SAT math score in 1996. Compare with the actual value of 508.

$$\begin{aligned} y &= 1.6(6) + 498 \\ y &= 9.6 + 498 \\ y &= 507.6 \approx 508 \end{aligned}$$

- d) Use the equation in (b) to predict the average SAT math score in 2006.

$$\begin{aligned} y &= 1.6(16) + 498 \\ y &= 25.6 + 498 = 523.6 \approx 524 \end{aligned}$$

For problems 26 – 41, solve the equation algebraically.

26. $3x - 4 = 6x + 5$

$$\begin{aligned} -9 &= 3x \\ \boxed{-3} &= \boxed{x} \end{aligned}$$

28. $2(5 - 2y) - 3(1 - y) = y + 1$

$$\begin{aligned} 10 - 4y - 3 + 3y &= y + 1 \\ 7 - y &= y + 1 \\ 6 &= 2y \\ \boxed{y} &= \boxed{3} \end{aligned}$$

30. $x^2 - 4x - 3 = 0$

$$x = 4 \pm \sqrt{16 + 12}$$

$$x = \frac{4 \pm \sqrt{28}}{2} = 2 \pm \sqrt{7}$$

32. $6x^2 + 7x = 3$

$$\begin{aligned} 6x^2 - 2x + 9x - 3 &= 0 \\ 2x(3x - 1) + 3(3x - 1) &= 0 \\ (2x + 3)(3x - 1) &= 0 \\ x &= -\frac{3}{2} \text{ or } \frac{1}{3} \end{aligned}$$

34. $x(2x + 5) = 4(x + 7)$

$$\begin{aligned} 2x^2 + 5x &= 4x + 28 \\ 2x^2 + x - 28 &= 0 \\ 2x(x + 4) - 7(x + 4) &= 0 \\ (2x - 7)(x + 4) &= 0 \\ x &= \frac{7}{2} \text{ or } -4 \end{aligned}$$

36. $4x^2 - 4x + 2 = 0$

$$\begin{aligned} 2(2x^2 - 2x + 1) &= 0 \\ x &= \frac{2 \pm \sqrt{4 - 8}}{4} = \frac{2 \pm 2i}{4} = \frac{1}{2} \pm \frac{i}{2} \end{aligned}$$

38. $x^2 = 3x$

$$\begin{aligned} x(x - 3) &= 0 \\ x &= 0 \text{ or } 3 \end{aligned}$$

40. $x^2 - 6x + 13 = 0$

$$\begin{aligned} x &= \frac{6 \pm \sqrt{36 - 52}}{2} \\ x &= \frac{6 \pm 4i}{2} = 3 \pm 2i \end{aligned}$$

27. $\frac{x-2}{3} + \frac{x+5}{2} = \frac{1}{3}$

$$\begin{aligned} 2x - 4 + 3x + 5 &= 2 \\ 5x + 1 &= 2 \\ 5x &= -9 \\ \boxed{x} &= \boxed{-\frac{9}{5}} \end{aligned}$$

29. $3(3x - 1)^2 = 21$

$$\begin{aligned} (3x - 1)^2 &= 7 \\ 9x^2 - 6x + 1 &= 7 \\ 9x^2 - 6x - 6 &= 0 \end{aligned}$$

31. $16x^2 - 24x + 7 = 0$

$$\begin{aligned} x &= \frac{24 \pm \sqrt{576 - 448}}{32} \\ x &= \frac{24 \pm \sqrt{128}}{32} = \frac{24 \pm 8\sqrt{2}}{32} = \frac{3 \pm \sqrt{2}}{4} \end{aligned}$$

33. $2x^2 + 8x = 0$

$$\begin{aligned} 2x(x + 4) &= 0 \\ x &= 0 \text{ or } -4 \end{aligned}$$

35. $|4x + 1| = 3$

$$\begin{aligned} 4x + 1 &= 3 & 4x + 1 &= -3 \\ 4x &= 2 & 4x &= -4 \\ x &= \frac{1}{2} & x &= -1 \end{aligned}$$

37. $-9x^2 + 12x - 4 = 0$

$$x = \frac{-12 \pm \sqrt{144 - 144}}{-18} = \frac{-12}{-18} = \frac{2}{3}$$

39. $4x^2 - 4x + 2 = 0$

$$\begin{aligned} 2(2x^2 - 2x + 1) &= 0 \\ x &= \frac{2 \pm \sqrt{4 - 8}}{4} = \frac{2 \pm 2i}{4} = \frac{1}{2} \pm \frac{i}{2} \end{aligned}$$

41. $x^2 - 2x + 4 = 0$

$$x = \frac{2 \pm \sqrt{4 - 16}}{2} = \frac{2 \pm 2i\sqrt{3}}{2} = 1 \pm i\sqrt{3}$$

42. Use completing the square to solve the equation $2x^2 - 3x - 1 = 0$.

$$2x^2 - 3x - 1 = 0$$

$$x^2 - \frac{3}{2}x - \frac{1}{2} = 0$$

$$x^2 - \frac{3}{2}x + \left(\frac{-3/4}{2}\right)^2 = \frac{1}{2} + \frac{9}{16}$$

$$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{17}{16}$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$$x - \frac{3}{4} = \pm \frac{\sqrt{17}}{4}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

43. Use the quadratic formula to solve the equation $3x^2 + 4x - 1 = 0$.

$$x = \frac{-4 \pm \sqrt{16 + 12}}{6} = \frac{-4 \pm 2\sqrt{7}}{6} = \frac{-2 \pm \sqrt{7}}{3}$$

Solve equations 44 – 47 graphically.

44. $3x^3 - 19x^2 - 14x = 0$

$$x = -\frac{2}{3}, 0, 7$$

45. $x^3 + 2x^2 - 4x - 8 = 0$

$$x = -2, x = 2$$

46. $x^3 - 2x^2 - 2 = 0$

$$x \approx 2.36$$

47. $|2x - 1| = 4 - x^2$

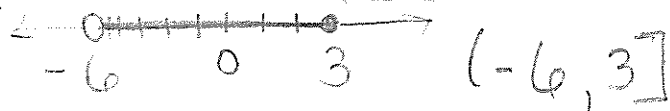
$$x = -1 \text{ and } x \approx 1.45$$

For problems 48 and 49, solve the inequality and draw a number line graph of the solution.

48. $-2 < x + 4 \leq 7$

$$-2 < x + 4 \implies x > -6$$

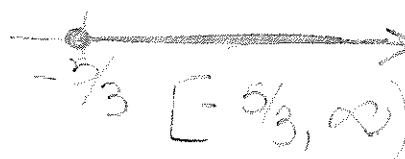
$$x + 4 \leq 7 \implies x \leq 3$$



49. $5x + 1 \geq 2x - 4$

$$3x \geq -5$$

$$x \geq -\frac{5}{3}$$



For problems 50 – 61, solve the inequality.

50. $\frac{3x - 5}{4} \leq \frac{1}{1}$

$$3x - 5 \leq 4$$

$$3x \leq 9$$

$$x \leq 3$$

$$3x - 5 \leq 4$$

$$3x \leq 9$$

$$x \leq 3$$

51. $|2x - 5| < 7$

$$2x - 5 < 7$$

$$2x < 12$$

$$x < 6$$

$$(-1, 6)$$

$$2x - 5 < -7$$

$$2x < -2$$

$$x < -1$$

52. $|3x + 4| \geq 2$

$$3x + 4 \geq 2 \implies 3x \geq -2 \implies x \geq -\frac{2}{3}$$

$$3x + 4 \leq -2 \implies 3x \leq -6 \implies x \leq -2$$

$$(-\infty, -2] \cup [-\frac{2}{3}, \infty)$$

53. $4x^2 + 3x > 10$

$$16 - 6 > 10$$

$$x < -2$$

$$x > \frac{5}{4}$$

$$(-\infty, -2) \cup (\frac{5}{4}, \infty)$$

$$-x^2 + 9x - 5x - 10 > 0$$

$$4x(x + 2) - 5(x + 2)$$

$$(4x - 5)(x + 2) > 0$$

$$x = \frac{5}{4}, x = -2$$

54. $2x^2 - 2x - 1 > 0$

$$x < -0.366 \quad (-\infty, 0.37) \cup (1.37, \infty)$$

$$x > 1.37$$

55. $9x^2 - 12x - 1 \leq 0$

graph

$$[-0.79, 1.41]$$

56. $x^2 - 9x \leq 3$
 $x^2 - 9x - 3 \leq 0$
 $x = \frac{9 \pm \sqrt{81 - 4(1)(-3)}}{2} = \frac{9 \pm \sqrt{93}}{2} \rightarrow 9.32, -0.32$
 $(-\infty, -0.32) \cup (9.32, \infty)$

57. $4x^3 - 9x + 2 > 0$ $(-1.6, 0.23) \cup (1.37, \infty)$
GRAPH

58. $\frac{|x+7|}{5} > 2$ $(-\infty, -17) \cup (3, \infty)$
 $|x+7| > -10$
 $x < -17$
 $x > 3$

59. $2x^2 + 3x - 35 < 0$ $(-5, 7/2)$
 $2x^2 + 10x - 7x - 35 = 0$
 $2x(x+5) - 7(x+5) = 0$
 $2x - 7 = 0$ $x + 5 = 0$
 $2x = 7$ $x = -5$
 $x = 7/2$ LOWER

60. $4x^2 + 12x + 9 \geq 0$ $x > 3/2$ or $x < 3/2$
 $(2x+3)^2 = 0$ SO
 $2x+3 = 0$ ALL REALS
 $2x = -3$
 $x = -3/2$

61. $x^2 - 6x + 9 < 0$ UPPER
 $(x-3)^2 = 0$ $(-\infty, 3)$
 $x - 3 = 0$
 $x = 3$

In problems 62 - 69, perform the indicated operation, and write the result in the standard form $a + bi$.

62. $(3 - 2i) + (-2 + 5i)$
 $1 + 3i$

63. $(5 - 7i) - (3 - 2i)$
 $2 - 5i$

64. $(1 + 2i)(3 - 2i)$
 $3 - 2i + 6i - 4i^2$
 $7 + 4i$

65. $(1 + i)^3$
 $(1 + 2i + i^2) = 2i(1 + i)$
 $2i + 2i^2$
 $-2 + 2i$

66. $(1 + 2i)^2(1 - 2i)^2$
 $(1 - 4i^2)(1 - 4i^2)$
 $(5)(5) = 25$

67. i^{29}
 i

68. $\sqrt{-16}$
 $4i$

69. $\frac{2+3i}{1-5i} \cdot \frac{1+5i}{1+5i}$ conjugate
 $\frac{2+3i+10i+15i^2}{1+25i^2} = \frac{-2-15+13i}{-24}$
 $= \frac{-13+13i}{24} = -\frac{1}{2} + \frac{1}{2}i$

70. A projectile is launched straight up from ground level with an initial velocity of 320 ft/sec.

a) When will the projectile's height above ground be 1538 ft?

$$h = -16t^2 + 320t$$

$$-16t^2 - 320t + 1538 = 0 \quad 8.03 < t < 12$$

$$t = \frac{320 \pm \sqrt{(-320)^2 - 4(16)(1538)}}{32}$$

b) When will the projectile's height above ground be at most 1538 ft?

$$[0, 8.03] \cup [12, \infty)$$

$$t = \frac{320 \pm 63}{32} = 12$$

c) When will the projectile's height above ground be greater than or equal to 1538 ft?

$$[8.03, 12]$$

$$t = 8.03$$

71. A commercial jet airplane climbs at takeoff with slope $m = \frac{4}{9}$. How far in the horizontal direction will the airplane fly to reach an altitude of 20,000 ft above the takeoff point?

$$\frac{\text{rise}}{\text{run}} = \frac{4}{9} = \frac{20,000}{x}$$

$$x = 45,000$$

$$4x = 180,000$$

72. Consider the collection of all rectangles that have length 1 cm more than three times their width, w .

a) Find the possible widths (in cm) of these rectangles if their perimeters are less than or equal to 150 cm.

$$l = 3w + 1$$

$$w = w$$

$$6w + 2 + 2w \leq 150$$

$$8w + 2 \leq 150$$

$$8w \leq 148$$

$$w \leq 18.5 \text{ cm}$$

b) Find the possible widths (in cm) of these rectangles if their areas are greater than 1500 cm².

$$3w^2 + w > 1500$$

$$3w^2 + w - 1500 = 0$$

$$-1 \pm \sqrt{1 - 18000}$$

$$6$$

$$-1 \pm 134.17$$

$$6$$

$$\frac{133.17}{6} = 22.19$$

$$w > 22.19$$