

Trig Identities

Name: _____ Period: _____

Trigonometric Identities

Students will learn to verify trigonometric identities

Precalculus/Honors

Precalculus

What are the reciprocal identities?

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\sin \theta = \frac{1}{\csc \theta} \quad \cos \theta = \frac{1}{\sec \theta} \quad \tan \theta = \frac{1}{\cot \theta}$$

What are the quotient identities?

(X, y)
(cos, sin)

Quotient Identities:

$$\frac{y}{x} = \tan \theta = \frac{\sin \theta}{\cos \theta} \quad \frac{x}{y} = \cot \theta = \frac{\cos \theta}{\sin \theta}$$

What are the Pythagorean identities?

Pythagorean Identities:

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

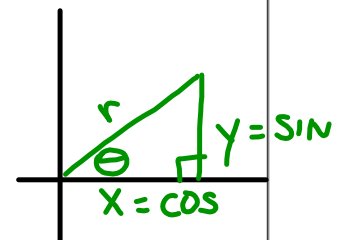
$$\tan^2 \theta = \sec^2 \theta - 1$$

$$1 = \sec^2 \theta - \tan^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

$$\cot^2 \theta = \csc^2 \theta - 1$$

$$1 = \csc^2 \theta - \cot^2 \theta$$



$$x^2 + y^2 = r^2$$

$$\cos^2 x + \sin^2 x = 1$$

What are the Odd- Even Identities

Even - Odd Identities

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x \quad \sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

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How do you verify identities?

Guidelines for Verifying Trigonometric Identities

- Start with the more complicated side, work step-by step to transform it to the simpler side.
- Look for opportunities to apply the fundamental identities.
- Use one or more of the following techniques:
 - Rewrite the more complicated side in terms of **sinx** and **cosx**.
 - Factor out the greatest common factor.
 - Separate a single-term quotient terms:

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$
 - Combine fractions using a common denominator.

Don't be afraid to stop and start over if you are not getting anywhere. Strategies leading to dead ends often provide clues to the solution.

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How do you verify an identity by replacing all terms with $\sin x$ or $\cos x$?

Example 1: Verify the expression.

$$\begin{aligned} (\csc x)(\tan x) &= \sec x \\ \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} &= \\ \frac{1}{\cos x} &= \sec x \checkmark \end{aligned}$$

Example 2: Verify the expression.

$$\begin{aligned} (\sin x)\tan x + (\cos x) &= \sec x \\ \left(\frac{\sin x}{1} \cdot \frac{\sin x}{\cos x}\right) + \frac{\cos x}{1} &= \frac{\sin^2 x}{\cos x} + \frac{\cos x}{1} \left(\frac{\cos x}{\cos x}\right) \\ \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} &= \frac{(\sin^2 x + \cos^2 x)}{\cos x} = \frac{1}{\cos x} = \sec x \checkmark \end{aligned}$$

How do you use factoring to verify an identity?

Example 3: Verify the expression.

$$\begin{aligned} x - xy^2 &= \\ \sin x - \sin x \cos^2 x &= \sin^3 x \\ \sin x(1 - \cos^2) &= \\ \sin x(\sin^2 x) &= \sin^3 x \end{aligned}$$

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How do you use multiple steps to verify an identity?

Example 4: Verify the identity.

$$\frac{1 + \sin x}{\cos x} = \sec x + \tan x$$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x \checkmark$$

How do you combine fractional expressions to verify an identity?

Example 5: Verify the identity.

$$\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} = 2 \csc x$$

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How do you verify an identity by multiplying the numerator and the denominator by the same factor?

Example 6: Verify the identity.

$$\frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

Example 7: Verify the identity.

$$\frac{\sec x + \csc(-x)}{\sec x \csc x} = \sin x - \cos x$$

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Verify each identity.

1. $\sin x \sec x = \tan x$

5. $\sin x \tan x \cos x = \sin^2 x$

2. $\cos x \csc x = \cot x$

6. $\sin x \cot x \cos x = \cos^2 x$

3. $\tan x \cos x = \sin x$

7. $\sec x - \sec x \sin^2 x = \cos x$

4. $\cot x \sin x = \cos x$

8. $\csc x - \csc x \cos^2 x = \sin x$

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9. $\cos^2 x - \sin^2 x = 1 - 2 \sin^2 x$

13. $\sin t \tan t = \frac{1 - \cos^2 t}{\cos t}$

10. $\csc \theta - \sin \theta = \cot \theta \cos \theta$

14. $\frac{\csc^2 t}{\cot t} = \csc t \sec t$

11. $\frac{\tan \theta \cot \theta}{\csc \theta} = \sin \theta$

15. $\cos x(\tan x + \sin x \cot x) = \sin x + \cos^2 x$

12. $\sin^2 x (1 + \cot^2 x) = 1$

16. $\frac{1 - \sin \theta}{\cos \theta} = \sec \theta - \tan \theta$

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$$17. \frac{\sin t}{\tan t} + \frac{\cos t}{\cot t} = \sin t + \cos t$$

$$21. (\sec \theta + 1)(\sec \theta - 1) = \tan^2 \theta$$

$$18. 1 - \frac{\sin^2 x}{1 + \cos x} = \cos x$$

$$22. \frac{(1 - \cos u)(1 + \cos u)}{\cos^2 u} = \tan^2 u$$

$$19. \csc^2 x \sec x = \sec x + \csc x \cot x$$

$$23. \frac{(1 + \tan^2 x)}{\sin^2 x + \cos^2 x} = \sec^2 x$$

$$20. \sin^2 \beta (1 + \cot^2 \beta) = 1$$

$$24. \tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$$

$$\begin{aligned} 7. \quad & \left(\frac{1}{\sin^2 A} \right) - \frac{1}{\tan^2 A} = \\ & \frac{1}{\sin^2 A} - \frac{\cot^2 A}{1} = \\ & \frac{1}{\sin^2 A} - \frac{\cos^2 A}{\sin^2 A} = \\ & \frac{(1 - \cos^2 A)}{\sin^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1 \end{aligned}$$

$$\begin{aligned} 8. \quad 1 - \frac{\sin^2 \theta}{\tan^2 \theta} &\rightarrow 1 - \left(\frac{\sin^2 \theta}{1} \cdot \frac{1}{\tan^2 \theta} \right) = \\ &1 - \frac{\sin^2 \theta}{1} \cdot \frac{\cot^2 \theta}{1} = 1 - \left[\frac{\cancel{\sin^2 \theta}}{1} \left(\frac{\cos^2 \theta}{\cancel{\sin^2 \theta}} \right) \right] \\ &= 1 - \cos^2 \theta = \sin^2 \theta \end{aligned}$$