

HPC 3/6/17

- Students will use the sum and difference formulas to find the values for trig functions that are not the standard angles found on the unit circle.
- Students will see their test scores.
- Students who have not taken the summative assessment on identities will give me the time that they will take the test either today or Tuesday.

Sum and Difference Formulas

Precalculus

Students will use the sum and difference formulas for sines, cosines, and tangents.

Sum and Difference Identities

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

When using the sum and difference formulas replace the variables u and v with angle measures found on the unit circle that will allow you to find the value of trig function of an angle not found on the unit circle.

Example 1: Using the Sine/Cosine of a sum or difference to find the exact value.

$$u = 60^\circ$$

$$v = 45^\circ$$

$$\begin{aligned} \text{a. } \cos 15^\circ &= \cos(60^\circ - 45^\circ) = \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ \\ &= \left(\frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \\ &= \frac{\sqrt{2} + \sqrt{6}}{4} \end{aligned}$$

$$u = 30^\circ$$

$$v = 45^\circ$$

$$\begin{aligned} \text{b. } \sin 75^\circ &= \sin(30^\circ + 45^\circ) = \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ \\ &= \left(\frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4} \end{aligned}$$

<p>Sum and Difference Formulas</p> <p><i>Students will use the sum and difference formulas for sines, cosines, and tangents.</i></p>	<p>Precalculus</p>
<p>$\frac{7\pi}{12} \cdot \frac{180^\circ}{\pi} = 105^\circ$</p> <p>$15^\circ$ $u = 45^\circ$ $v = 60^\circ$</p> <p>$u = 80^\circ$ $v = 20^\circ$</p> <p>$u = 25^\circ$ $v = 20^\circ$</p>	<p>Example 2: Using the Sine/Cosine of a sum or difference to find the exact value.</p> <p>a. $\sin \frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin \frac{\pi}{4} \cos \frac{\pi}{3} + \cos \frac{\pi}{4} \sin \frac{\pi}{3}$ $= \left(\frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$ $= \frac{\sqrt{2} + \sqrt{6}}{4}$</p> <p>b. $\tan \frac{\pi}{12} = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\tan \frac{\pi}{4} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{4} \tan \frac{\pi}{6}} = \frac{1 - \sqrt{3}}{1 + (1)(\sqrt{3})} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$</p> <p>Example 3: Write each expression as the sine, cosine or tangent of an angle. Then find the exact value of the expression.</p> <p>a. $\cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ = \cos(80^\circ - 20^\circ) = \cos 60^\circ = \frac{1}{2}$</p> <p>b. $\cos 25^\circ \cos 20^\circ - \sin 25^\circ \sin 20^\circ = \cos(25^\circ + 20^\circ) = \cos 45^\circ = \frac{\sqrt{2}}{2}$</p>

Sum and Difference Formulas

Name: _____ Date: _____

Find the exact value of each expression.

1. $\cos 105^\circ$

2. $\sin 15^\circ$

3. $\cos 75^\circ$

4. $\sin 255^\circ$

5. $\cos 285^\circ$

6. $\cos 195^\circ$

7. $\tan 105^\circ$

8. $\tan 195^\circ$

9. $\tan 165^\circ$

Write each expression as the sine, cosine or tangent of an angle. Then find the exact value of the expression.

10. $\sin 25^\circ \cos 5^\circ + \cos 25^\circ \sin 5^\circ$

11. $\cos 105^\circ \cos 45^\circ + \sin 105^\circ \sin 45^\circ$

12. $\frac{\tan 10^\circ + \tan 35^\circ}{1 - \tan 10^\circ \tan 35^\circ}$

13. $\sin \frac{5\pi}{12} \cos \frac{\pi}{4} - \cos \frac{5\pi}{12} \sin \frac{\pi}{4}$

14. $\frac{\tan \frac{\pi}{5} - \tan \frac{\pi}{80}}{1 + \tan \frac{\pi}{5} \tan \frac{\pi}{80}}$