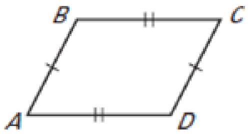
8.3 Show that a Quadrilateral is a Parallelogram

Goal: Use properties to identify parallelograms.

THEOREM 8.7

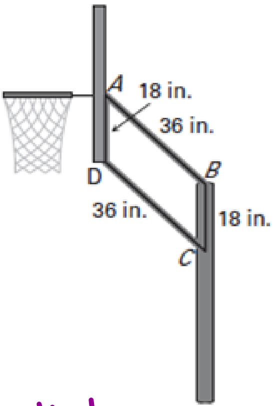
If both pairs of opposite Sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If $\overline{AB} \cong \overline{DC}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Example 1 *Solve a real-world problem*

Basketball In the diagram at the right, $\overline{AB}$ and $\overline{DC}$ represent adjustable supports of a basketball hoop. *Explain* why $\overline{AD}$ is always parallel to $\overline{BC}$.



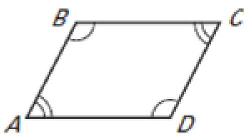
$\overline{AB} \cong \overline{DC}$ and

$\overline{AD} \cong \overline{BC}$

$\therefore$ ABCD is a parallelogram

THEOREM 8.8

If both pairs of opposite ∠'s of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If $\angle A \cong$ ∠C and $\angle B \cong$ ∠D, then *ABCD* is a parallelogram.

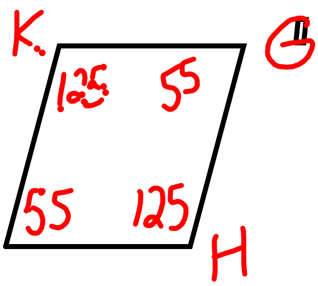
Checkpoint **Complete the following exercise**

1. In quadrilateral *GHJK*, $m\angle G = 55°$, $m\angle H = 125°$, and $m\angle J = 55°$. Find $m\angle K$. What theorem can you use to show that *GHJK* is a parallelogram?

$\angle G \cong \angle J$

$\angle H \cong \angle K$

$\therefore$ GHJK is parallelogram



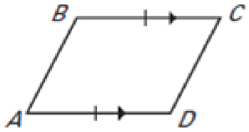
$m\angle K = 360 - (55+55+125)$

$= 360 - 235$

$= 125$

THEOREM 8.9

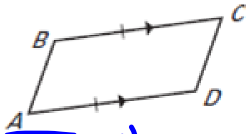
If ~~one~~ same pair of opposite sides of a quadrilateral are ___$\cong$___ and ___$\parallel$___, then the quadrilateral is a parallelogram.



If $\overline{BC}$ _$\cong$_ $\overline{AD}$ and $\overline{BC}$ _$\parallel$_ $\overline{AD}$, then $ABCD$ is a parallelogram.

Example 2 ***Identify a parallelogram***

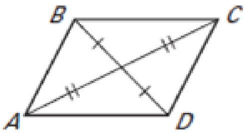
Lights The headlights of a car have the shape shown at the right. *Explain* how you know that $\angle B \cong \angle D$.



If $\overline{BC}$ is both $\parallel$ and $\cong$ $\overline{AD}$ then ABCD is a parallelogram and $\angle B \cong \angle D$

THEOREM 8.10

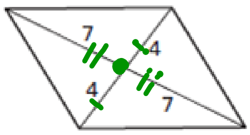
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



If $\overline{BD}$ and $\overline{AC}$ bisect each other, then $ABCD$ is a parallelogram.

Checkpoint

2. What theorem can you use to show that the quadrilateral is a parallelogram?

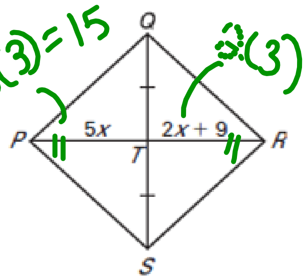


8.10

Example 3 *Use algebra with parallelograms*

For what value of x is quadrilateral $PQRS$ a parallelogram?

$5(3)=15$

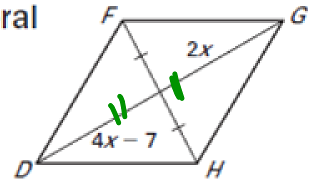


$2(3)+9=15$

$$5x = 2x + 9$$
$$-2x \quad -2x$$
$$3x = 9$$
$$\frac{3x}{3} = \frac{9}{3}$$
$$x = 3$$

Checkpoint

3. For what value of x is quadrilateral *DFGH* a parallelogram?



$$2x = 4x - 7$$
$$-4x \quad -4x$$
$$-2x = -7$$
$$\frac{-2x}{-2} = \frac{-7}{-2}$$
$$x = 3.5$$

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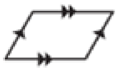
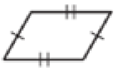
CONCEPT SUMMARY: WAYS TO PROVE A QUADRILATERAL IS A PARALLELOGRAM

1. Show both pairs of opposite sides are parallel. **(Definition)**

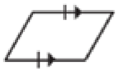


2. Show both pairs of opposite sides are congruent. **(Theorem 8.7)**



3. Show both pairs of opposite angles are congruent. **(Theorem 8.8)**

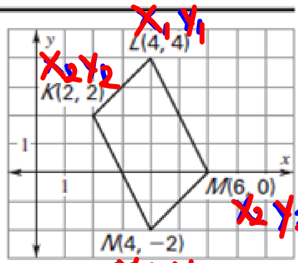
4. Show one pair of opposite sides are congruent and parallel. **(Theorem 8.9)**



5. Show the diagonals bisect each other. **(Theorem 8.10)**

Example 4 *Use coordinate geometry*

Show that quadrilateral *KLMN* is a parallelogram.



$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{LM} = \frac{0-4}{6-4} = \frac{-4}{+2} = -2$$

$$m_{KN} = \frac{-2-2}{4-2} = \frac{-4}{2} = -2$$

$$m_{LK} = \frac{2-4}{2-4} = \frac{-2}{-2} = 1$$

$$m_{MN} = \frac{0+2}{6-4} = \frac{2}{2} = 1$$

∴ KLMN is a parallelogram because opp. sides have the same slope so they are //.

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