

8.2 Use Properties of Parallelograms

Goal: Find angle and side measures of parallelograms.

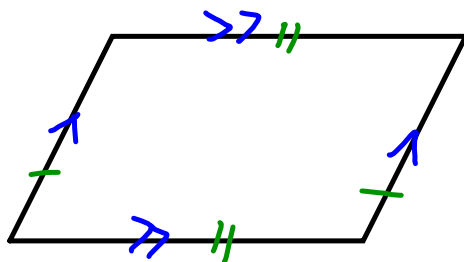
Vocabulary

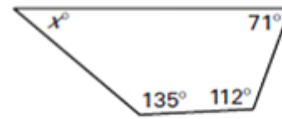
Polygon: A closed plane figure with 3 or more sides. Each side intersects exactly two other sides at each endpoint.

Quadrilateral: A polygon with four sides whose interior angles sum to 360° .

Diagonal: A segment that joins two non-consecutive vertices of a polygon.

Parallelogram: A quadrilateral with both pairs of opposite sides both parallel and congruent.



Example 1 Find an unknown interior angle measureFind the value of x in the diagram shown.

* The sum of the interior \angle 's of a Quadrilateral (4 sides) = 360°

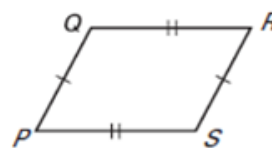
$$135^\circ + 112^\circ + 71^\circ + x = 360^\circ$$

$$\begin{array}{r} 318^\circ + x = 360^\circ \\ - 318^\circ \quad - 318^\circ \\ \hline x = 42^\circ \end{array}$$

THEOREM 8.3

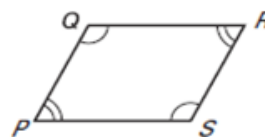
If a quadrilateral is a parallelogram,
then its opposite sides are congruent.

If $PQRS$ is a parallelogram, then
 $\overline{QP} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$.

**THEOREM 8.4**

If a quadrilateral is a parallelogram,
then its opposite angles are congruent.

If $PQRS$ is a parallelogram, then
 $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

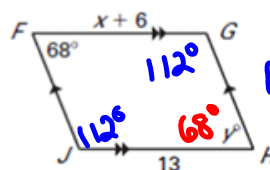


Example 2 Use properties of parallelogramsFind the values of x and y .

$$\overline{FG} \cong \overline{JH}$$

$$x+6 = 13$$

$$\begin{array}{r} x+6 = 13 \\ -6 \quad -6 \\ \hline x = 7 \end{array}$$



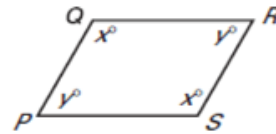
$$m\angle G = 180 - 68^\circ$$

$$m\angle G = 112^\circ$$

$$\begin{array}{l} \angle H \cong \angle F \\ y = 68^\circ \end{array}$$

THEOREM 8.5

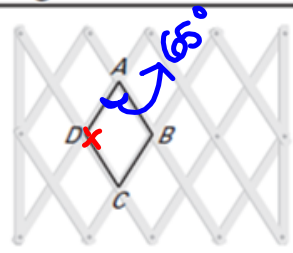
If a quadrilateral is a parallelogram,
then its consecutive angles are
Supplementary.



If $PQRS$ is a parallelogram, then $x^\circ + y^\circ = \underline{180^\circ}$.

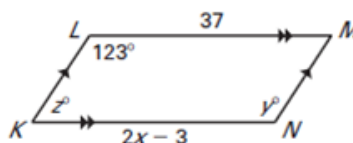
Example 3 Use properties of a parallelogram

Gates As shown, a gate contains several parallelograms. Find $m\angle ADC$ when $m\angle DAB = 65^\circ$.



$$\begin{aligned}
 x + 65 &= 180 \\
 x &= 180 - 65 = 115^\circ \\
 m\angle ADC &= 115^\circ
 \end{aligned}$$

✓ **Checkpoint** Find the indicated measure in $\square KLMN$ shown at the right.



$$\begin{array}{r} 2x - 3 = 37 \\ +3 \quad +3 \\ \hline \end{array}$$

$$\begin{array}{r} 2x = 40 \\ \frac{2}{2} \quad \frac{2}{2} \\ \hline \end{array}$$

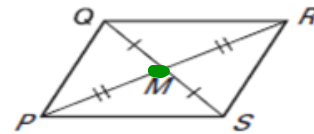
$$x = 20$$

$$\begin{array}{l} \angle L \cong \angle N \\ 123^\circ = y \end{array}$$

$$\begin{array}{l} z = 180^\circ - 123^\circ \\ z = 57^\circ \end{array}$$

THEOREM 8.6

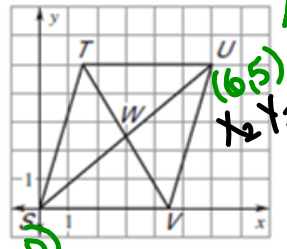
If a quadrilateral is a parallelogram, then its diagonals bisect each other.



$$\overline{QM} \cong \overline{MS} \text{ and } \overline{PM} \cong \overline{MR}$$

Example 4 Use properties of a parallelogram

The diagonals of $\square STUV$ intersect at point W . Find the coordinates of W .



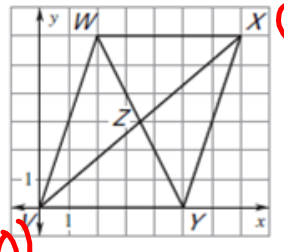
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$W = \left(\frac{0+6}{2}, \frac{0+5}{2} \right)$$

$$W = (3, 2.5)$$

✔ **Checkpoint** Complete the following exercises.

4. The diagonals of $\square VWXY$ intersect at point Z . Find the coordinates of Z .

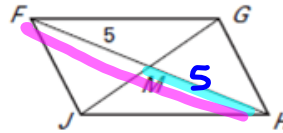


$$Z = \left(\frac{0+7}{2}, \frac{0+6}{2} \right)$$

$$Z = (3.5, 3)$$

$(0,0)$
 x_1, y_1

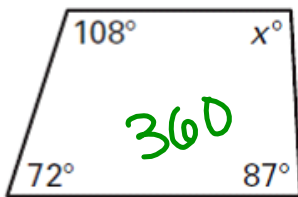
5. Given that $\square FGHI$ is a parallelogram, find MH and FH .



$$\begin{array}{c} \overline{MH} \\ \parallel \\ 5 \end{array} \quad \begin{array}{c} \overline{FH} \\ \parallel \\ 10 \end{array}$$

Find the value of x .

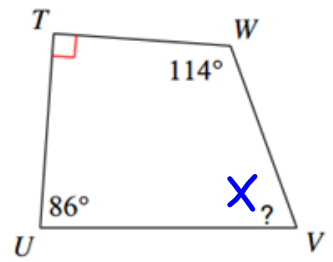
6.



$$x + 108 + 72 + 87 = 360$$

$$x = 93^\circ$$

7. Find the $m\angle V$

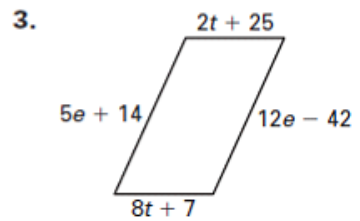
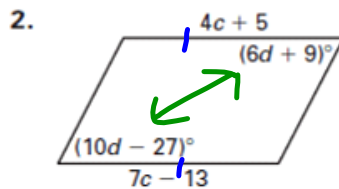
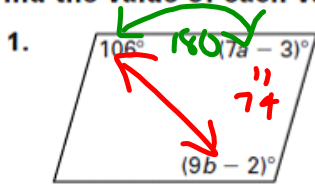


$$x + 86 + 90 + 114 = 360$$

$$x = 70^\circ$$

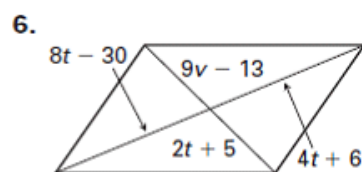
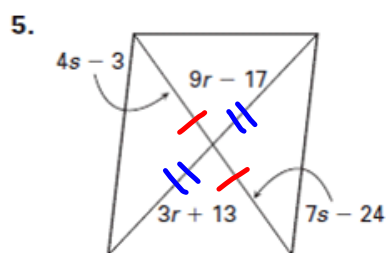
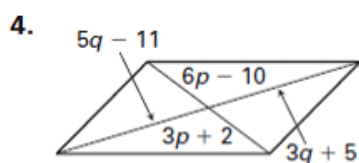
LESSON 8.2 Practice C
For use with pages 514-521

Find the value of each variable in the parallelogram.



$$\begin{aligned}
 7c - 13 &= 4c + 5 \\
 -4c & \quad -4c \\
 \hline
 3c - 13 &= 5 \\
 +13 & \quad +13 \\
 \hline
 3c &= 18 \\
 c &= 6
 \end{aligned}$$

$$\begin{aligned}
 10d - 27 &= 6d + 9 \\
 -6d & \quad -6d \\
 \hline
 4d - 27 &= 9 \\
 4d &= 36 \\
 d &= 9
 \end{aligned}$$

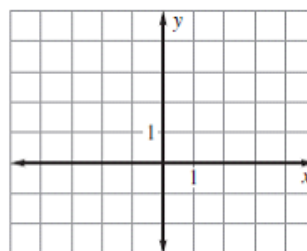


$$4s - 3 = 7s - 24$$

$$9r - 17 = 3r + 13$$

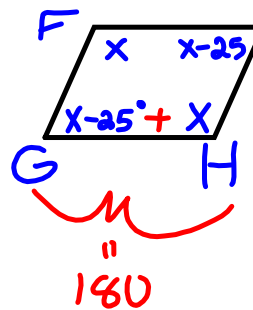
7. The coordinates for $\square ABCD$ are $A(-1, 3)$, $B(4, 2)$, $C(2, -1)$, and $D(-3, 0)$. Plot the points and draw $\square ABCD$ on the coordinate plane. Then draw the diagonals \overline{AC} and \overline{BD} . Label the intersection of the diagonals as point E . What are the coordinates of point E ?

ex 9



8. In $\square WXYZ$, $m\angle W$ is 50 degrees more than $m\angle X$. Sketch $\square WXYZ$. Find the measure of each interior angle. Then label each angle with its measure.

9. In $\square EFGH$, $m\angle G$ is 25 degrees less than $m\angle H$. Sketch $\square EFGH$. Find the measure of each interior angle. Then label each angle with its measure.



77.5°
 $\angle H = 102.5$
 $\angle G = 102.5 - 25$
 $x + x - 25 = 180$
 $2x - 25 = 180$
 $\quad + 25 \quad + 25$

 $2x = 205$
 $x = 102.5$

Find the indicated measure in $\square ABCD$.

10. $m\angle AEB$

12. $m\angle AED$

14. $m\angle BAD$

16. $m\angle ADC$

11. $m\angle BAE$

13. $m\angle ECB$

15. $m\angle DCE$

17. $m\angle DCB$

