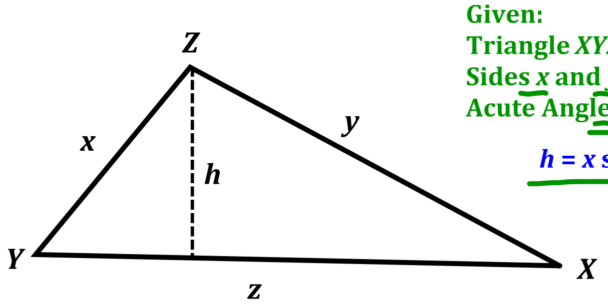
Law of Sines: Ambiguous Case (SSA)

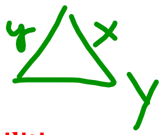
Another method for determining the number of solutions (triangles) by comparing the given side lengths to the triangle's height.



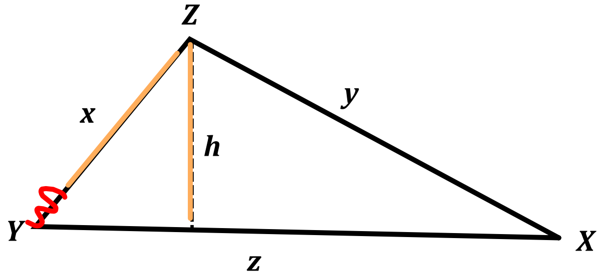
Given:
Triangle XYZ and
Sides x and y and
Acute Angle Y

$h = x \sin Y$

Four possibilities:
- $x < h$
- $x = h$
- $x > y$
- $h < x < y$



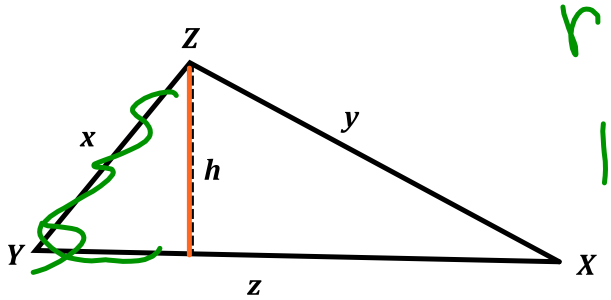
Possibility One: $x < h$



no solution

$\sin X > 1.0$

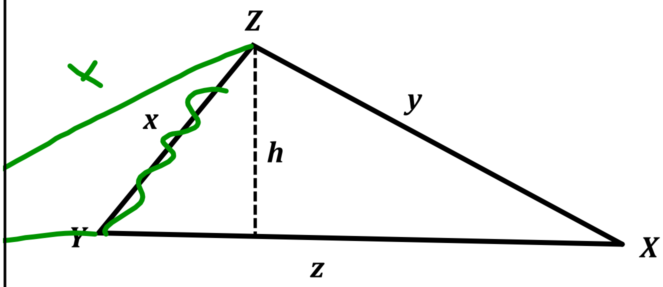
Possibility Two: $x = h$



right Δ

1 triangle

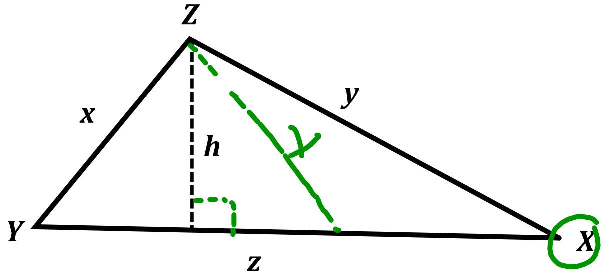
Possibility Three: $x > y$



If $x > y$
then $\angle Z$ is obtuse
on $\triangle$

Possibility Four: $h < x < y$

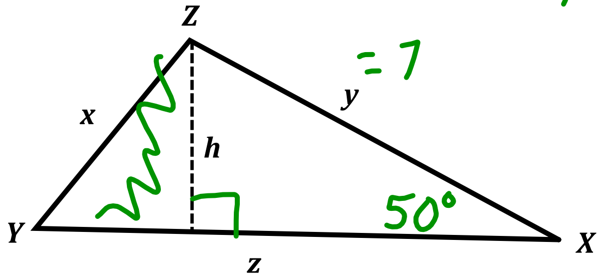
2 solution



Possible lengths for side *x* (given *y* = 7 and *X* = 50°)

$h < y$

$5.36 < 7$



$h = y \sin X$

$h = 7 \sin 50° \approx 5.36$

One Triangle:

$x \geq 7$

Two Triangles:

$5.36 < x < 7$

No Triangles:

$x < 5.36$

The Ambiguous case: How Many Triangles?

Given: A, a, b: $h = b \sin A$

If A is obtuse and:

- $a \leq b$; No Triangle
- $a > b$; One Triangle
- Never two triangles :)

If A is acute and:

- $a < h$; No Triangle
- $a = h$ or $a > b$; One Triangle
- $h < a < b$; Two Triangles

How many Triangles?

1. $A = 36^0$; $a = 2$; $b = 7$

$h = b \sin A$

$h = 7 \sin 36$

$h = 4.11$

$h > a$

$4.11 > 2$

NO Δ

2. $C = 30^0$; $a = 18$; $c = 9$

$h = a \sin C$

$h = 18 \sin 30^\circ$

$h = 9$

$h = c$

One Rt Δ

3. $B = 82^0$; $b = 7$; $c = 15$

acute

$h = c \sin B$

$h = 14.85$

$b < h$

no Δ

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What could be the length of side *c*?

acute

1. $C = 43^0$; $b = 12$

$h = 12 \sin 43 \approx 8.18$

One Triangle:

$c \geq 12$

Two Triangles:

$h < c < b$

$8.18 < c < 12$

No Triangles:

$c < 8.18$

2. $C = 82^0$; $b = 18$

$h =$

One Triangle:

Two Triangles:

No Triangles: