HPC/RPC Name KEY

Sequences and Series Review

Some formulas you may find useful . . .				
$a_n = a_1 + (n-1)d$	$a_n = a_1 \cdot r^{n-1}$	$\sum a_n = \left(\frac{a_1 + a_n}{2}\right)n$	$\sum a_n = \frac{a_1(1-r^n)}{1-r}$	$\sum a_n = \frac{a_1}{1-r}$

1. Find the 21st term of the sequence 16, 8, 4, . . . GEOMETRIC: $a_n = a_1 r^{n-1}$

$r = \frac{1}{2}$, $a_1 = 16$: $a_{21} = 16\left(\frac{1}{2}\right)^{21-1} = 16\left(\frac{1}{2}\right)^{20} = 16\left(\frac{1}{1048576}\right) \approx \boxed{0.000015}$

2. Determine the 19th term of the sequence -2, 1, 4, 7, . . . ARITHMETIC: $a_n = a_1 + (n-1)d$

$d = 3$, $a_1 = -2$, $n = 19$: $a_{19} = -2 + (19-1)(3) = -2 + (18)(3) = -2 + 54 = \boxed{52}$

3. The fifteenth and thirtieth terms of an arithmetic sequence are –27 and –102, respectively. Find the common difference, the first term and the Explicit Rule for the sequence.

$-27 = a_1 + (15-1)d \Rightarrow -27 = a_1 + 14d$

$-102 = a_1 + (30-1)d \Rightarrow -(-102 = a_1 + 29d)$

$75 = -15d$

$\boxed{-5 = d}$

OR

$d = \frac{-102-(-27)}{30-15} = \frac{-75}{15} = -5$

$-27 = a_1 + (14)(-5)$

$-27 = a_1 - 70$

$43 = a_1$

$a_1 = 43$ $\quad$ $d = -5$ $\quad$ so: $a_n = 43 + (n-1)(-5)$

$= 43 - 5n + 5$

$= -5n + 48$

4. The fourth and tenth terms of a geometric sequence are 3 and 192, respectively. Find the common ratio, the first term and the Explicit Rule for the sequence.

$a_4 = 3 = a_1 r^{(4-1)} \Rightarrow 3 = a_1 r^3 \Rightarrow a_1 = \frac{3}{r^3}$

$a_{10} = 192 = a_1 r^{(10-1)} \Rightarrow 192 = a_1 r^9 \Rightarrow a_1 = \frac{192}{r^9}$

so: $\frac{3}{r^3} = \frac{192}{r^9}$

$\frac{3r^9}{r^3} = \frac{192 r^9}{r^3}$

$\frac{3r^6}{3} = \frac{192}{3}$

$r^6 = 64$

$\boxed{r = 2}$

a_1:

$3 = a_1 (2)^3$

$3 = a_1 \cdot 8$

$\boxed{a_1 = \frac{3}{8}}$

Rule:

$a_n = \frac{3}{8}(2)^{n-1}$

For Questions 5 – 10;

a) Determine whether the infinite sequence is arithmetic or geometric,

b) Determine whether the sequence converges or diverges, and

c) If it converges, find the limit/sum (what it converges to).

5. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \frac{1}{2^n}, \ldots$ $\boxed{\text{GEOMETRIC}}$

$r = \frac{1}{2}$: $\boxed{\text{CONVERGES}}$: $\left|\frac{1}{2}\right| < 1$ ∴

$$\Sigma = \frac{a_1}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = \boxed{2}$$

6. 2, 5, 8, 11, . . . ARITHMETIC

SEQUENCE DIVERGES

NO INFINITE SUM POSSIBLE

7. $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, \ldots$ $\boxed{\text{GEOMETRIC}}$ $r = 3$

SEQUENCE DIVERGES

$|3| > 1$ ∴

8. $3n - 1$ ARITHMETIC:

SEQUENCE DIVERGES

9. $1(0.5)^{n-1}$ $\boxed{\text{GEOMETRIC}}$ $r = 0.5$, SO SEQUENCE CONVERGES:

CONVERGES TO: $\frac{a_1}{1-r} = \frac{1}{1-0.5} = \frac{1}{0.5} = \boxed{2}$

10. $6(-0.9)^n$ GEOMETRIC:

$$\frac{a_1}{1-r} = \frac{6}{1-(-0.9)} = \frac{6}{1.9} \approx 3.158$$

CONVERGES TO ≈ 3.158

11. Given the sequence 1, 7, 13, . . . $d = 6$ $a_1 = 1$

a. Write the Explicit Formula that represents this sequence.

$$a_n = 1 + (n-1)6 = 1 + 6n - 6 = \boxed{6n - 5 = a_n}$$

b. Find the 7th, 12th and 55th terms of the sequence.

$a_7 = 6(7) - 5 = 42 - 5 = 37$ / $a_{12} = 6(12) - 5 = 72 - 5 = 67$ / $a_{55} = 6(55) - 5 = 330 - 5 = 325$

c. Find the sum of the finite sequence if it has 55 terms.

$$\sum_{n=1}^{55} 6n - 5 = \left(\frac{1 + 325}{2}\right)55 = (163)55 = \boxed{8965}$$

$\left(\frac{a_1 + a_n}{2}\right)n$ $a_1 = 1$ $a_n = 325$ $n = 55$

d. If possible, find the sum of the sequence if it is infinite.

NOT POSSIBLE: ARITHMETIC INFINITE SUMS DON'T EXIST

12. Given the sequence 125, 25, 5, . . . $r = \frac{1}{5}$ $a_1 = 125$

a. Write the Explicit Formula that represents this sequence.

$$a_n = 125\left(\frac{1}{5}\right)^{n-1}$$

b. Find the 7th and 15th terms of the sequence.

$$a_7 = 125\left(\frac{1}{5}\right)^{7-1} = 125\left(\frac{1}{5}\right)^6 = \frac{1}{125} = 0.008$$

$$a_{15} = 125\left(\frac{1}{5}\right)^{15-1} = 125\left(\frac{1}{5}\right)^{14} = \frac{1}{48828125} \approx 2.048 \times 10^{-8}$$

c. Find the sum of the finite sequence if it has 21 terms.

$$\Sigma = \frac{a_1(1-r^n)}{1-r} \qquad \sum_{n=1}^{21} 125\left(\frac{1}{5}\right)^{n-1} = \frac{125\left(1-\left(\frac{1}{5}\right)^{21}\right)}{1-\frac{1}{5}} \approx \boxed{156.25}$$

d. If possible, find the sum of the sequence if it is infinite. $\frac{a_1}{1-r}$

$$\sum_{n=1}^{\infty} 125\left(\frac{1}{5}\right)^{n-1} = \frac{125}{1-\frac{1}{5}} = \frac{125}{\frac{4}{5}} = \frac{125(5)}{4} = \frac{625}{4} = \boxed{156.25}$$

13. Jacob is planning a trapezoid shaped patio that has 21 rows. His plan calls for 10 blocks in the first row and 60 in the last row. How many blocks does Jacob need to buy for this project?

Sum: $S = \left(\frac{a_1 + a_n}{2}\right)n$

$$\left.\begin{array}{l} a_1 = 10 \\ a_n = 60 \\ n = 21 \end{array}\right\} \text{Sum} = \left(\frac{10+60}{2}\right)21 = (35)21 = \boxed{735 \text{ BLOCKS}}$$

14. In his piggy bank, Bingo dropped \$1.00 on May 1, \$1.75 on May 2, \$2.50 on May 3 and so on until the last day of May.

a) How much did he drop in his piggy bank on May 19? ARITHMETIC: $d = 0.75$, $a_1 = 1.00$

$$a_n = 1.00 + (n-1)(0.75) = 1.00 + 0.75n - 0.75 = 0.25 + 0.75n$$

$$a_{19} = 0.25 + 0.75(19) = 0.25 + 14.25 = \$14^{50}$$

b) What was his total deposit in his piggy bank for the month of May?

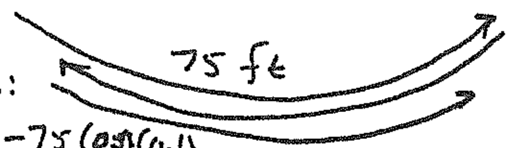
MAY HAS 31 DAYS, SO

$$a_{31} = 0.25 + 0.75(31) = 0.25 + 23.25 = \$23.50 \text{ ON THE LAST DAY OF MAY}$$

$$\text{TOTAL} = \left(\frac{\$1.00 + \$23.50}{2}\right)31 = (\$12.25)31 = \boxed{\$379.75}$$

20. Tarzan, while swinging from vine to vine in the jungle, misses a vine and has to swing back and forth on his vine until he comes to a complete stop. If he travels 75 feet on his initial swing and each subsequent swing is 10% smaller, how many total feet does Tarzan travel until his swinging stops?

SWING 1:



SWING 2: 75 ft − 75(0.1) ft

SWING 3: 75 ft + (75(0.1)) − 75(0.9)(0.1) .... etc

THIS IS A GEOMETRIC SEQUENCE:

$a_1 = 75\text{ ft}$

$r = 0.9$ → EACH SWING IS 90% THE SIZE OF THE PREVIOUS ONE.

SINCE NO "n" IS AVAILABLE (YOU AREN'T TOLD HOW MANY TIMES TARZAN SWINGS), THE SUM IS CONSIDERED INFINITE: and $\Sigma = \frac{a_1}{1-r}$

THE SEQUENCE IS: $a_n = 75(0.9)^{n-1}$

$$\sum_{n=1}^{\infty} 75(0.9)^{n-1} = \frac{75}{1-0.9} = \frac{75}{0.1} = \frac{75}{\frac{1}{10}} = 750\text{ ft}$$

TARZAN SWINGS ABOUT 750 FEET BEFORE STOPPING.