Graphs of Rational Functions

- *Students will be able to analyze rational functions and sketch their graphs.*

RPC/HPC

What is an rational function?

If $N(x)$ and $D(x)$ are functions with $D(x) \neq 0$,

$$R(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x^1 + a_0 x^0}{b_n x^n + b_{n-1}x^{n-1} + \ldots + b_1 x^1 + b_0 x^0}$$ is a **Rational Function**

Sketch the graph of $f(x) = \frac{1}{x}$. (Reciprocal)

State the Domain and Range.

x	2	1	0.5	0.1	0.01	0.001	$\to 0$
$f(x)$	0.5	1	2	10	100	1000	und.

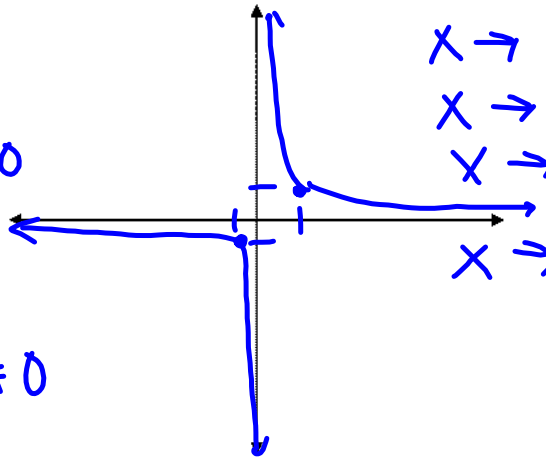
x	-2	-1	-0.5	-0.1	-0.01	-0.001	$\to 0$
$f(x)$	-0.5	-1	-2	-10	-100	-1000	und.

What is the Domain of f(x)?

$(-\infty, \infty)$ $x \neq 0$

What is the Range of f(x)?

$(-\infty, \infty)$ $y \neq 0$



$x \to -\infty$ $f(x) \to 0^-$

$x \to 0^-$ $f(x) \to -\infty$

$x \to 0^+$ $f(x) \to +\infty$

$x \to \infty$ $f(x) \to 0^+$

Section 2.6: Graphs of Rational Functions

- *Students will be able to analyze rational functions and sketch their graphs.*

Honors PreCalculus

Can we find asymptotes of rational functions without graphing?

Two basic types of asymptotes:

- **Vertical**
  - **Indicates a restriction on the domain of a function**
  - $\lim_{x \to a} f(x) = \infty$ or $\lim_{x \to a} f(x) = -\infty$
- **End Behavior: Can be horizontal, slant or a non-linear function**
  - **Indicates a restriction on the range of a function**
  - $\lim_{x \to \infty} f(x) = b$ or $\lim_{x \to -\infty} f(x) = b$

$\frac{1}{x}$

n = 0
m = 1

Finding asymptotes:

Vertical: Occur at zeros of the denominator

End Behavior: Compare the degrees of the numerator (n) and denominator (m)

- **If n < m, the asymptote is horizontal; $y = 0$**
- **If n = m, the asymptote is horizontal; $y = \frac{a_n}{b_m}$**

$\frac{3x^2}{4x^2}$ $y = 3/4$

- **If n > m, end behavior asymptote is found by actually dividing the polynomials**

x-intercepts (a.k.a. "solutions") occur at the zeros of the numerator

x-intercepts = roots = zeros = solutions.

Example 1: Find any asymptotes and the Domain for $f(x)=\frac{4}{(x-2)^3}$

Vertical Asymptote(s):

$x-2=0$

$x=2$

So $x\neq 2$

Domain:

$(-\infty, 2)\cup(2,\infty)$

End Behavior Asymptote(s):

$n=0 \quad m=3 \quad n<m \quad y=0$ horiz. Asym

Sep 28-9:11 AM

Example 2: Find any asymptotes and the Domain for $f(x)=\frac{2x^2}{x+1}$

Vertical Asymptote(s):

$x = -1$

Domain:

$(-\infty, -1) \cup (-1, \infty)$

$$x+1 \overline{)\,2x^2}$$

$2x - 2$ r $\frac{2}{x+1}$

$2x^2 + 2x$

$-2x$

$-2x - 2$

2

End Behavior Asymptote(s):

$n = 2 \quad m = 1 \quad n > m$

Slant asymptote $y = 2x - 2$

Sep 28-9:11 AM

Example 3: Find any asymptotes and the Domain for $f(x)=\frac{2x^2}{(x^2-4)}$

Vertical Asymptote(s):

$x^2-4=0$

$x^2=4$

$x=\pm 2$

Domain:

$(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

End Behavior Asymptote(s):

$n=2$ $m=2$ $n=m$ $y=\frac{2}{1}=2$

Horizontal

Sep 28-9:11 AM

Example 3: Find any asymptotes and the Domain for $f(x)=\frac{x^3-3x^2+3x+1}{x-1}$

Vertical Asymptote(s): $x=1$

Domain: $(-\infty,1)\cup(1,\infty)$

End Behavior Asymptote(s):

$n=3 \quad m=1 \quad n>m$

$y = x^2-2x+1$

parabolic asymptote

$$x-1\overline{)\,x^3-3x^2+3x+1}$$

x^2-2x+1 R $\frac{+2}{x-1}$

$-(x^3-x^2)$

$-2x^2+3x+1$

$-(-2x^2+2x)$

$x+1$

$x-1$

$+2$

Example 1: Find any asymptotes and the Domain for $f(x)=\frac{4}{(x-2)^3}$

Vertical Asymptote(s): **Domain**:

End Behavior Asymptote(s):

Example 2: Find any asymptotes and the Domain for $f(x)=\frac{2x^2}{x+1}$

Vertical Asymptote(s): **Domain**:

End Behavior Asymptote(s):

Example 3: Find any asymptotes and the Domain for $f(x)=\frac{2x^2}{(x^2-4)}$

Vertical Asymptote(s): **Domain**:

End Behavior Asymptote(s):

Example 4: Find any asymptotes and the Domain for $f(x)=\frac{x^3-3x^2+3x+1}{x-1}$

Vertical Asymptote(s): **Domain**:

End Behavior Asymptote(s):

HPC/RPC: Graphs of Rational Functions **Name** _____

Find any asymptotes (Vertical or End Behavior) and the Domain of the following functions:

1. $f(x)=\frac{1}{x+3}$

2. $f(x)=\frac{2x^2-1}{x^2+3}$

3. $f(x)=\frac{x-3}{x^2+3x}$

4. $f(x)=\frac{x-2}{x^2-2x-3}$

5. $f(x)=\frac{-3x^2+x+12}{x^2-4}$

6. $f(x)=\frac{2x+1}{x^2-x}$

7. $f(x)=\frac{x^2-x-2}{x^2-2x-8}$

8. $f(x)=\frac{2x^5+x^2-x+1}{x^2-1}$