

HPC/RPC Graphing Rational Functions NOTES

Name _____

Date _____ Per _____

The steps given below are going to help with graphing rational functions.

- 1) [y=0] and [x=0] (if there are any).
- 2) Find the vertical asymptote(s) by setting the denominator equal to 0 and solving.
- 3) Identify any holes in the graph (factors in the denominator that reduce out).
- 4) Find the end behavior asymptote (if it exists).
- 5) Graph asymptotes using dashed lines.
- 6) Pick some points that you can plot to get the general shape of the graph.

$$\frac{(x-1)(x/3)}{x(x+3)}$$

$$x = -3$$

Example 1: Graph the function and identify all key features of the graph.

$$\frac{1}{x} f(x) = \frac{4}{x-2} = \frac{4}{1} \left(\frac{1}{x-2} \right)$$

1) x-intercept(s)? NA y-intercept? -2

$$\frac{0}{1} = \frac{4}{(x-2)}$$

$$0 \neq 4$$

$$f(0) = \frac{4}{0-2} = -2$$

2) Vertical Asymptote(s) x=2

$$x-2=0 \quad x=2$$

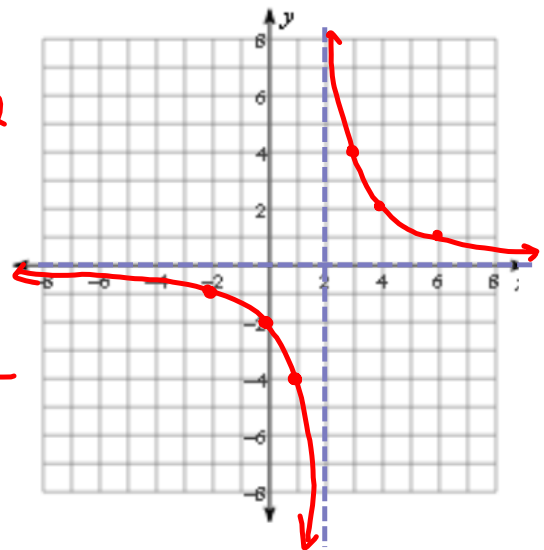
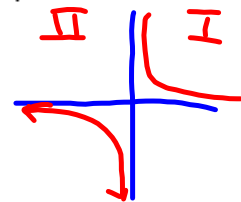
$$\frac{4}{x-2}$$

$$\frac{0}{-2}$$

3) Holes? NONE

4) End Behavior Asymptote y=0

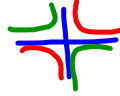
$$n=0 \quad m=1 \quad n < m$$



Example 2: Graph the function and identify all key features of the graph.

$\frac{1}{x}$

$$f(x) = -\frac{2}{x-1} + 1$$



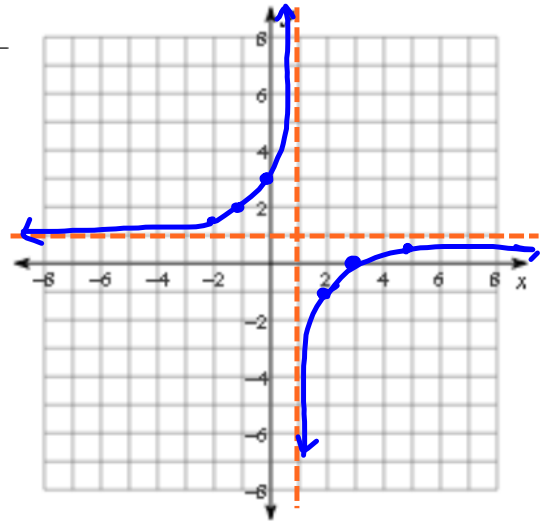
1) x-intercept(s)? 3 y-intercept? 3

$$0 = -\frac{2}{x-1} + 1 \quad \begin{matrix} -2 = -x + 1 \\ -3 = -x \end{matrix}$$

$$\frac{-1}{1} = \frac{-2}{(x-1)}$$

2) Vertical Asymptote(s) $x=1$

3) Holes? NONE



4) End Behavior Asymptote $y=1$

$$N=0 \quad M=1 \quad n < m \quad y=0$$

Example 3: Graph the function and identify all key features of the graph.

$$f(x) = \frac{3}{x^2+x-2} = \frac{3}{(x+2)(x-1)} = \frac{3}{(1.5)(-1.5)} = \frac{3}{-2.25}$$

1) x-intercept(s)? NA y-intercept? -1.5

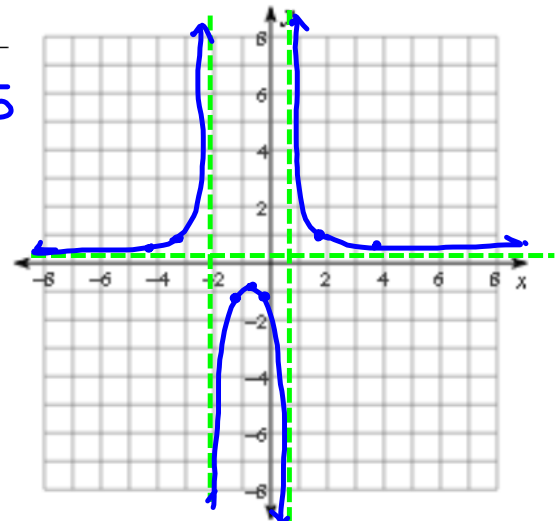
$$\frac{0}{1} = \frac{3}{(x^2+x-2)}$$

$$0 \neq 3$$

$$f(0) = \frac{3}{-2} = -1.5$$

2) Vertical Asymptote(s) $x=-2, 1$

3) Holes? NA



4) End Behavior Asymptote $y=0$
 $n=0 \quad m=2 \quad n < m$

$16 - 20 +$

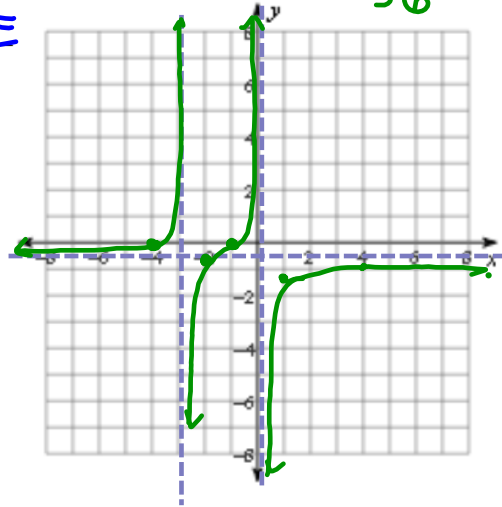
Example 4: Graph the function and identify all key features of the graph.

$f(x) = \frac{x^2 + 5x + 4}{-2x^2 - 6x}$ $\frac{4 - 10 + 4}{-8 + 12} = \frac{-2}{4}$ $f(0) = \frac{0}{0}$

1) x-intercept(s)? $-4, -1$ y-intercept? **NONE**

2) Vertical Asymptote(s) $x = 0, -3$

3) Holes? **NONE**



4) End Behavior Asymptote $y = -\frac{1}{2}$
 $n = 2$ $m = 2$ $n = m$

Example 5: Graph the function and identify all key features of the graph.

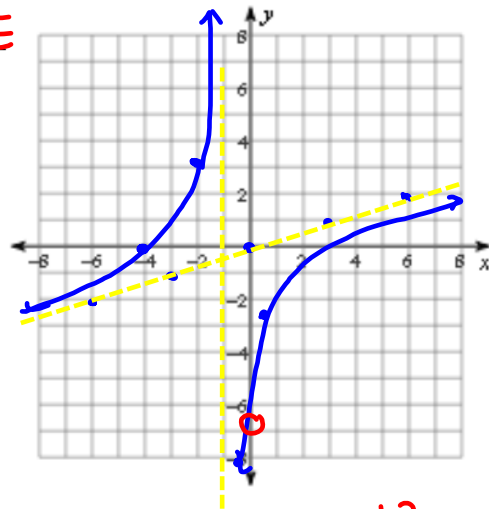
$f(x) = \frac{x^3 + x^2 - 12x}{3x^2 + 3x}$ $\frac{x(x+4)(x-3)}{3x(x+1)}$ $f(0) = \frac{0}{0}$ und.

1) x-intercept(s)? $x = -4, +3$ y-intercept? **NONE**

$\frac{-8 + 4 + 24}{12 - 6} = \frac{20}{6}$

2) Vertical Asymptote(s) $x = -1$

3) Holes? $x = 0$
 pt. of discontinuity



4) End Behavior Asymptote $y = +\frac{1}{3}x$
 $n = 3$ $m = 2$ $n > m$

$\frac{1}{3}x$ r $\frac{-12x}{3x^2 + 3x}$
 $3x^2 + 3x \overline{) x^3 + x^2 - 12x + 0}$
 $\underline{x^3 + x^2}$
 $-12x$