The Dot Product

Students will find the dot product, find the angle between two vectors and determine if vectors are orthogonal.

The dot product of two vectors results in a **number**, *not a vector.*

If $\mathbf{v} = \langle a_1, b_1 \rangle$ and $\mathbf{w} = \langle a_2, b_2 \rangle$, then the dot product is:

$$\mathbf{v} \cdot \mathbf{w} = a_1 a_2 + b_1 b_2$$

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Example 1: *Find the Dot Product.*

v = <5, – 2> w = <-3, 4>

a. v•w = $5 \cdot (-3) + (-2)(4) = -23$

b. w•v = $-3(5) + 4(-2) = -23$

c. v•v = $5 \cdot 5 + (-2)(-2) = 29$

v = <7, 4> w = <2 ,– 1>

a. v•w = $7 \cdot 2 + 4(-1) = 10$

b. w•v = $2 \cdot 7 + (-1)(4) = 10$

c. w•w = $2 \cdot 2 + (-1)(-1) = 5$

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Properties of the Dot Product

1. $u \bullet v = v \bullet u$
2. $u \bullet (v+w) = u \bullet v + u \bullet w$
3. $0 \bullet v = 0$
4. $v \bullet v = ||v||^2$
5. $(cu) \bullet v = c(u \bullet v) = u \bullet (CV)$

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Finding the Angle between two Vectors

$$\cos\theta = \frac{v \bullet w}{\|v\|\|w\|} \qquad \text{so} \qquad \theta = \cos^{-1}\left(\frac{v \bullet w}{\|v\|\|w\|}\right)$$

Example 2: Find the angle between the two vectors.

a. v = <3 , 2> w = <1, 4>

$v \bullet w = 3 \cdot 1 + 2 \cdot 4 = 11$

$\|v\| = \sqrt{3^2+2^2} = \sqrt{13}$

$\|w\| = \sqrt{1^2+4^2} = \sqrt{17}$

$\theta = \cos^{-1}\left(\frac{11}{\sqrt{13}\cdot\sqrt{17}}\right) = \cos^{-1}\left(\frac{11}{\sqrt{221}}\right)$

$= 42.27°$

b. v = <4, 3> w = <1, 2>

$v \bullet w = 4 \cdot 1 + 3 \cdot 2 = 10$

$\|v\| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

$\|w\| = \sqrt{1^2+2^2} = \sqrt{5}$

$\theta = \cos^{-1}\left(\frac{10}{5\sqrt{5}}\right) = 26.57°$

Determining if Vectors Are Parallel, Orthogonal or Neither.

"orthogonal" means perpendicular

To determine if vectors are parallel, orthogonal or neither find the angle between the two vectors.

θ	Parallel, Orthogonal, or Neither
0°	Parallel
180°	Parallel
90°	Orthogonal
other	Neither

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Example 3: Determine if the vectors are Parallel, Orthogonal or Neither.

a. $v = \langle 6, -3 \rangle$ $w = \langle 1, 2 \rangle$

$v \cdot w = 6 \cdot 1 + (-3)2 = 0$

$\|v\| = \sqrt{6^2 + (-3)^2} = \sqrt{45}$

$\|w\| = \sqrt{1^2 + 2^2} = \sqrt{5}$

$\theta = \cos^{-1}\left(\frac{0}{\sqrt{45}\sqrt{5}}\right) = 90$

orthogonal

b. $v = \langle 4, -3 \rangle$ $w = \langle -8, 6 \rangle$

$v \cdot w = 4(-8) + (-3)(6) = -50$

$\|v\| = \sqrt{4^2 + (-3)^2} = 5$

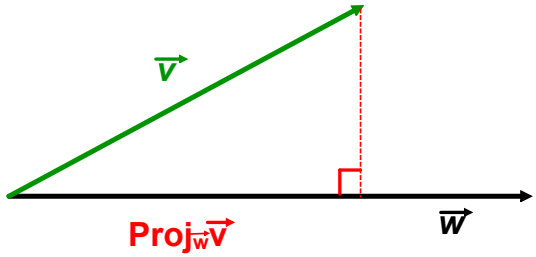
$\|w\| = \sqrt{(-8)^2 + (6)^2} = 10$

$\theta = \cos^{-1}\left(\frac{-50}{5 \cdot 10}\right) = 180^\circ$

// but opp direction

Projection of a vector onto another vector

Projection of $\vec{v}$ onto $\vec{w}$



$$\cos\theta = \frac{\|\text{proj}_w v\|}{\|v\|}$$

$$\|\text{proj}_w v\| = \|v\|\cos\theta$$

$$\theta = \cos^{-1}\frac{v \cdot w}{\|v\|\|w\|}$$

$$\cos\theta = \frac{v \cdot w}{\|v\|\|w\|}$$

$$\|v\|\cos\theta = \frac{v \cdot w}{\|w\|}$$

$$\|\text{Proj}_w v\| = \frac{v \cdot w}{\|w\|} \cdot \frac{w}{\|w\|}$$

$$= \left(\frac{v \cdot w}{\|w\|^2}\right) \times w$$

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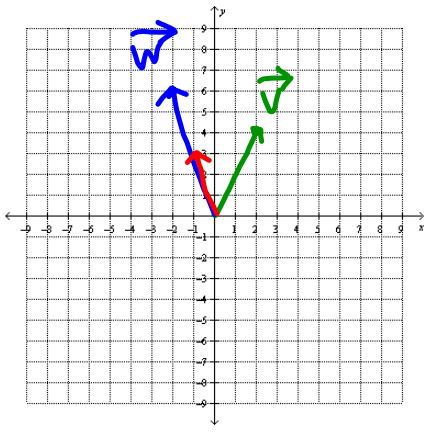
Example 4: Find the vector projection of **v** onto **w**.

v = <2,4> **w** = <-2,6>



$$\text{Proj}_w v = \frac{20}{(\sqrt{40})^2}\langle -2, 6\rangle$$

$$= \frac{1}{2}\langle -2, 6\rangle$$

$$= \langle -1, 3\rangle$$

$$v \cdot w = (2)(-2) + (4)(6) = 20$$

$$\|w\| = \sqrt{(-2)^2 + (6)^2} = \sqrt{40}$$

Definition of Work

$W = \vec{F} \cdot \cos\theta \cdot \vec{d}$

$d = \vec{v}$

$W = \|F\| \|\vec{v}\| \cos\theta$

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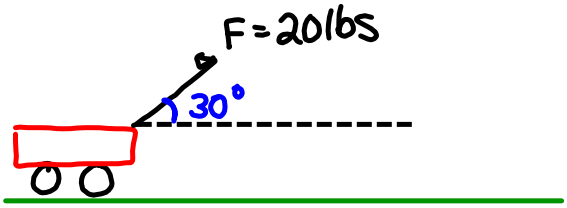
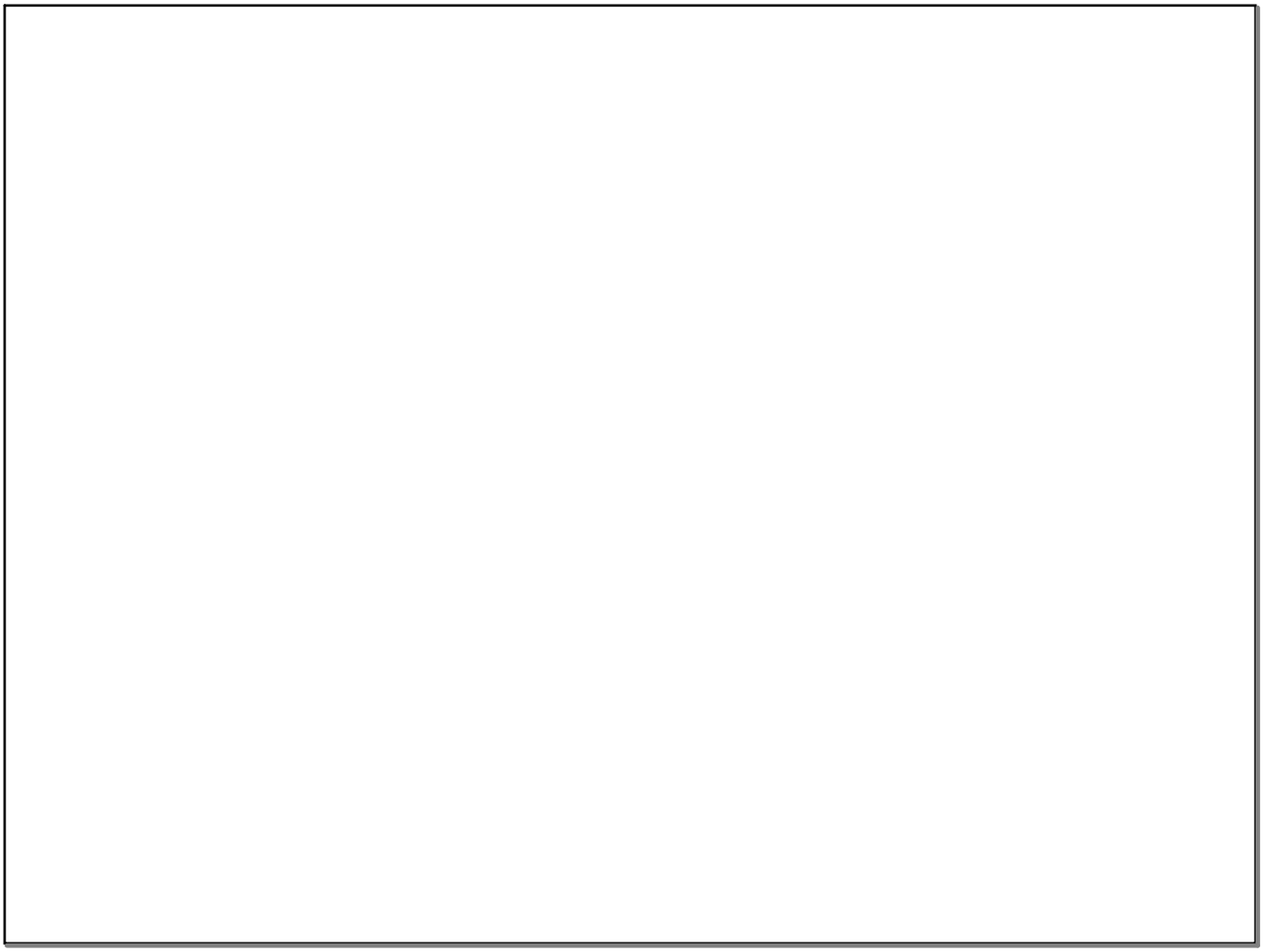
Example 6: A child pulls a wagon along level ground by exerting a force of 20 pounds on the handle that makes a 30° angle with the horizontal. How much work is done pulling the wagon 150 feet?



$\|\vec{F}\| = 20$ lbs
$\|\vec{v}\| = 150$ ft
$\theta = 30°$

$W = 20 \cdot 150 \cdot \cos 30°$
$= 2598$ ft/lbs



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