

Name: \_\_\_\_\_

Date: \_\_\_\_\_ Per. : \_\_\_\_\_

## Precalculus

## Solving Trigonometric Equations

## Review

Level 1

1. Find the exact value of each expression.

$$\begin{aligned} \text{a. } \sin 195^\circ &= \sin(150^\circ + 45^\circ) \\ &= \sin 150 \cos 45 + \cos 150 \sin 45 \\ &= \left( +\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \right) + \left( -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} \right) = \frac{+\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \text{b. } \tan 15^\circ &= \tan(45^\circ - 30^\circ) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}} \\ &= \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \end{aligned}$$

$$\begin{aligned} \text{c. } \cos \frac{5\pi}{12} &= \cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) = \cos \frac{3\pi}{4} \cos \frac{\pi}{3} - \sin \frac{3\pi}{4} \sin \frac{\pi}{3} \\ &= \left( -\frac{\sqrt{2}}{2} \cdot \frac{1}{2} \right) + \left( \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} \right) = -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{-\sqrt{2} + \sqrt{6}}{4} \end{aligned}$$

2. Write the expression as the tangent of an angle, then find the exact value.

$$\text{a. } \frac{\tan 155 + \tan 85}{1 - \tan 155 \tan 85} = \tan(155 + 85) = \tan 240^\circ = \frac{\sqrt{3}}{1}$$

$$\text{b. } \sin \frac{15\pi}{36} \cos \frac{\pi}{6} - \cos \frac{15\pi}{36} \sin \frac{\pi}{6} = \sin\left(\frac{15\pi}{36} - \frac{\pi}{6}\right) = \sin \frac{9\pi}{36} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\text{c. } \cos \frac{2\pi}{3} \cos \frac{\pi}{6} + \sin \frac{2\pi}{3} \sin \frac{\pi}{6} = \cos\left(\frac{2\pi}{3} - \frac{\pi}{6}\right) = \cos \frac{3\pi}{6} = \cos \frac{\pi}{2} = 0$$

Honors Precalculus  
Solving Trigonometric Equations  
Review  
Level 2

3. Solve the trigonometric equation for all values of  $x$  over the interval  $[0, 2\pi]$

a.  $3 \tan x + 6 = 0$

$$3 \tan x = -6$$

$$\tan x = -2$$

No solution

b.  $5 \cot \theta = \sqrt{3} + 4 \cot \theta$

$$\cot \theta = \sqrt{3}$$

$$\cot \theta = \frac{\sqrt{3}}{1}$$

$$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$$

c.  $12 \sin^2 x - 3 = 6 \sin^2 x$

$$6 \sin^2 x = 3$$

$$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$$

$$\sin x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

d.  $4 \cos^2 x - 3 = 0$

$$4 \cos^2 x = 3$$

$$\sqrt{\cos^2 x} = \sqrt{\frac{3}{4}}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

e.  $4 \tan^2 x - 1 = \tan^2 x$

$$3 \tan^2 x = 1$$

$$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$$

$$\tan x = \pm \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

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Honors Precalculus  
Solving Trigonometric Equations  
Review  
Level 3

4. Solve the trigonometric equation for all values of x.

a.  $2\tan^2\theta \sin\theta = \frac{2}{\cot^2\theta}$

$$2\tan^2\theta \sin\theta = 2\tan^2\theta$$

$$2\tan^2\theta(\sin\theta - 1) = 0$$

$$2\tan^2\theta = 0 \quad \sin\theta - 1 = 0$$

$$\tan^2\theta = 0 \quad \sin\theta = 1$$

$$\tan\theta = 0$$

$$\theta = 0, \frac{\pi}{2}, \pi \} + 2n\pi$$

b.  $\sec^2\theta - 1 = 2\tan\theta - 1$

$$1 + \tan^2\theta - 1 - 2\tan\theta + 1 = 0$$

$$\tan^2\theta - 2\tan\theta + 1 = 0$$

$$(\tan\theta - 1)^2 = 0$$

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4} \} + 2n\pi$$

c.  $6\cos 2\theta - 1 = 4\cos 2\theta$

$$2\cos 2\theta = 1$$

$$\cos 2\theta = \frac{1}{2}$$

$$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6} + 2n\pi$$

d.  $\csc^2 x - \cot x \csc^2 x = 0$

$$\csc^2 x (1 - \cot x) = 0$$

$$\csc^2 x = 0 \quad \cot x = 1$$

$$\csc x = 0 \quad \tan x = 1$$

$$\sin x = \frac{1}{0}$$

$$\text{und} \quad x = \frac{\pi}{4}, \frac{5\pi}{4} \} + 2n\pi$$

Honors Precalculus  
 Solving Trigonometric Equations  
 Review

e.  $\cot^2 x - \sec x \cot^2 x = 0$

$$\cot^2 x (1 - \sec x) = 0$$

$$\cot^2 x = 0 \quad 1 - \sec x = 0$$

$$\cot x = 0 \quad \sec x = 1$$

$$\tan x \text{ und } \cos x = 1$$

$$x = \cancel{0 + 2n\pi} \quad 0, \frac{\pi}{2}, \frac{3\pi}{2} + 2n\pi$$

f.  $1 - \cos^2 \theta = \sin \theta + 2$

$$\sin^2 \theta - \sin \theta - 2 = 0$$

$$(\sin \theta - 2)(\sin \theta + 1)$$

$$\sin \theta = 2 \quad \sin \theta = -1$$

NO sol'n

$$\theta = \frac{3\pi}{2} + 2n\pi$$

g.  $2\sin^4 x - 3\sin^2 x = -1$

$$(2\sin^2 x - 1)(\sin^2 x - 1)$$

$$\sin^2 x = \frac{1}{2} \quad \sin^2 x = 1$$

$$\sin x = \pm \frac{\sqrt{2}}{2}, \pm 1$$

$$x = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2} \right\} + 2n\pi$$