

Conics Day 1

Name: \_\_\_\_\_

Completing the Square

What is the purpose of completing the square:

The purpose of completing the square is to either factor a prime quadratic equation or to more easily graph a conic section.

Example 1: Solve the following quadratic equations by finding the square root.

a.  $(x^2 - 8x + 16) = 25$

$$\sqrt{(x-4)^2} = \sqrt{5^2}$$

$$x-4 = \pm 5$$

$$x-4 = -5$$

$$x = -1$$

$$x-4 = 5$$

$$x = 9$$

b.  $2x^2 - 24x + 8 = 0$

S1:  $x^2 - 12x + 4 = 0$

S2:  $x^2 - 12x = -4$

S3:  $x^2 - 12x + 36 = -4 + 36$   
 $x^2 - 12x + 36 = 32$

S4:  $(x-6)^2 = 32$

S5:  $\sqrt{(x-6)^2} = \sqrt{32}$        $\sqrt{16}\sqrt{2}$   
 $x-6 = \pm 4\sqrt{2}$

S6:  $x-6 = -4\sqrt{2}$        $x-6 = 4\sqrt{2}$   
 $x = 6 - 4\sqrt{2}$        $x = 6 + 4\sqrt{2}$

When an equation is not a square you will sometimes need to add a term to make the expression a square in order to solve it. Consider a trinomial in general form:  $Ax^2 + Bx + C = 0$

The steps to make a trinomial a square are listed below:

Step 1: Divide the equation by A so that the leading coefficient is 1

Step 2: Subtract C from both sides of the equation

Step 3: Divide B by 2 and square the result. Then add to both sides of the equation.  $\left(\frac{B}{2}\right)^2$

Step 4: Factor the left side of the equation.

Step 5: Take the square root of both sides of the equation.

Step 6: Solve for x

You Try:

a.  $x^2 - 8x - 128 = 0$

S2  $x^2 - 8x = 128$   
 S3  $x^2 - 8x + 16 = 128 + 16$   
 $x^2 - 8x + 16 = 144$   
 S4  $(x - 4)^2 = 12^2$   
 S5  $\sqrt{(x-4)^2} = \sqrt{12^2}$   
 S6:  $x - 4 = \pm 12$   
 $x = -8, 16$

b.  $3x^2 - 36x - 144 = 0$

S1  $x^2 - 12x - 48 = 0$   
 S2  $x^2 - 12x = 48$   
 S3  $x^2 - 12x + 36 = 84$   
 S4  $(x - 6)^2 = 84$   
 S5  $\sqrt{(x-6)^2} = \sqrt{84} = \sqrt{4 \cdot 21}$   
 S6  $x - 6 = \pm 2\sqrt{21}$   
 $x = 6 - 2\sqrt{21}$  and  $6 + 2\sqrt{21}$

Conic Sections can also be written in general form:  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$  and can be written in standard form. Completing the square is part of process to go from general form to standard form.

Example 2: Write the standard form equation of the conic section.

a.  $4x^2 + y^2 - 8x - 8 = 0$

$(4x^2 - 8x) + y^2 = 8$   
 $4(x^2 - 2x + 1) + y^2 = 8 + 4$   
 $4(x - 1)^2 + y^2 = 12$   
 $\frac{4(x-1)^2}{12} + \frac{y^2}{12} = \frac{12}{12}$   
 $\frac{(x-1)^2}{3} + \frac{y^2}{12} = 1$

b.  $16x^2 - 9y^2 - 96x + 36y - 36 = 0$

$16x^2 - 96x - 9y^2 + 36y = 36$   
 $16(x^2 - 6x + 9) - 9(y^2 - 4y + 4) = 36 + 144 - 36$   
 $16(x - 3)^2 - 9(y - 2)^2 = 144$   
 $\frac{16(x-3)^2}{144} - \frac{9(y-2)^2}{144} = \frac{144}{144}$   
 $\frac{(x-3)^2}{9} - \frac{(y-2)^2}{16} = 1$

c.  $y^2 - 4y - 2x + 6 = 0$

d.  $x^2 - 16x - 8y + 80 = 0$

$x^2 - 16x = 8y - 80$   
 $x^2 - 16x + 64 = 8y - 80 + 64$   
 $(x - 8)^2 = 8(y - 2)$