

## HPC January 19, 2017

**Learning Objective:** *Students will be able to graph angles in standard position and calculate positive and negative coterminal angles.*

- Return Unit Circle Formative Assessments - if you need to retake setup a date and time with me.
- Special Right Triangle and Unit Circle D3 Assignment Questions.
- Notes and practice problems for Angle Measures and Trigonometric Functions.
- Special Right Triangle and Unit Circle D3 check in if time

## Unit Circle Day 3 WS answers

- |                          |                   |                           |                  |                            |                           |
|--------------------------|-------------------|---------------------------|------------------|----------------------------|---------------------------|
| 1. $\frac{2\sqrt{3}}{3}$ | 2. 0              | 3. $-\frac{2\sqrt{3}}{3}$ | 4. $\frac{1}{2}$ | 5. 2                       | 6. $-\frac{\sqrt{3}}{2}$  |
| 7. $\frac{1}{2}$         | 8. $-\frac{1}{2}$ | 9. 2                      | 10. Und.         | 11. 1                      | 12. $\sqrt{2}$            |
| 13. $\sqrt{2}$           | 14. $-\sqrt{3}$   | 15. 1                     | 16. $\sqrt{2}$   | 17. $-\frac{2\sqrt{3}}{3}$ | 18. $-\frac{\sqrt{3}}{3}$ |

19) Find all angles (in radians)  $\theta$  on the unit circle that satisfy the expression  $\sin \theta = -\sqrt{3}/2$

$4\pi/3, 5\pi/3$

20) Find all angles (in radians)  $\theta$  on the unit circle that satisfy the expression  $\csc \theta = \sqrt{2}$

$\pi/4, 3\pi/4$        $\sin = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = \frac{1}{\sin}$

21) Find all angles (in radians)  $\theta$  on the unit circle that satisfy the expression  $\tan \theta = \sqrt{3}$

$\pi/6, 7\pi/6$

22) Find all angles (in radians)  $\theta$  on the unit circle that satisfy the expression  $\sec \theta = \text{undefined}$      $\pi/2, 3\pi/2$

14.  $\tan \frac{5\pi}{3}$   
 $(\frac{1}{2}, -\frac{\sqrt{3}}{2})$

$\tan = \frac{y}{x} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{1} = -\sqrt{3}$

**Angle Measure and Trigonometric Functions**

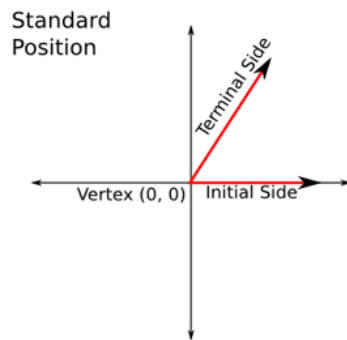
- Students will place angles in standard position and differentiate between positive and negative angle rotations

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**Standard position of non-acute angles**

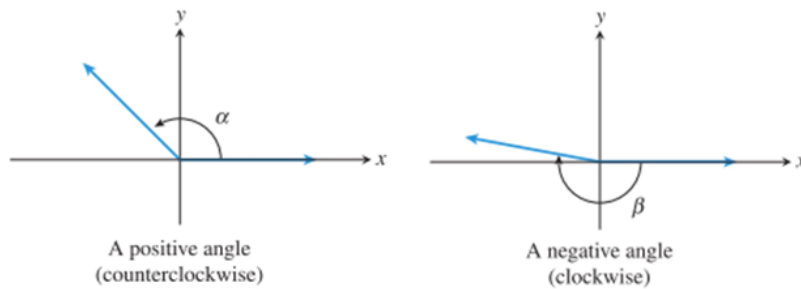
Standard position of an angle on the coordinate plane:

- Initial side on the positive  $x$  axis
- Terminal side can be in any quadrant



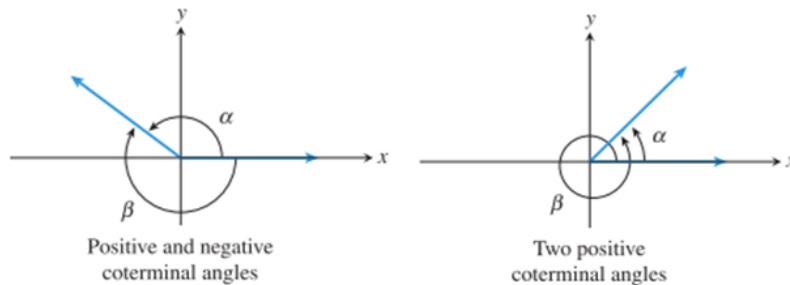
**Rotation of Angles**

Angle Rotation: Positive vs. Negative



**Coterminal angles**

Coterminal angles: Angles that have the same initial side and the same terminal side, but different measures.



Angle Measure and Trigonometric Functions

- Students will find coterminal angles

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**Example 1:** Sketch the angle in Standard Position, then find values for (and sketch) one positive and one negative coterminal angle for each of the following:

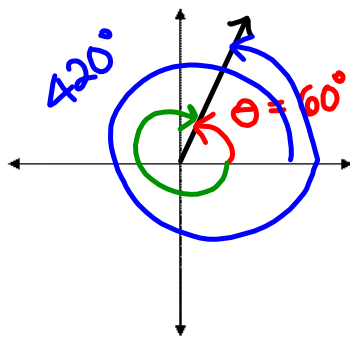
+cot  
 $60^\circ + 360^\circ$   
 $420^\circ$   
 -cot  
 $60^\circ - 360^\circ$   
 $-300^\circ$

$\Delta - -410$

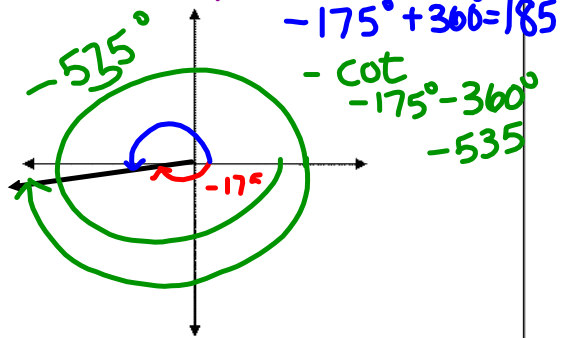
+cot  
 $\frac{4\pi}{7} + 2\pi(\frac{2}{7})\pi$   
 $\frac{4\pi}{7} + \frac{14\pi}{7}$   
 $\frac{18\pi}{7}$

-cot  
 $\frac{4\pi}{7} - \frac{14\pi}{7}$   
 $-\frac{10\pi}{7}$

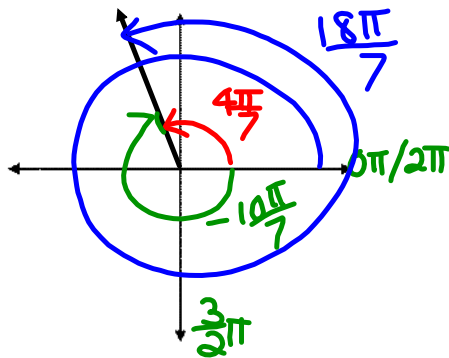
a.  $60^\circ$  Q I



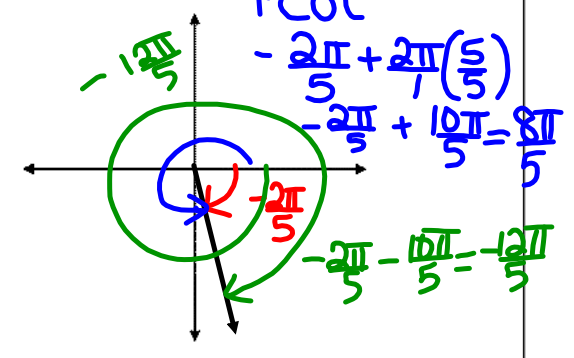
b.  $-175^\circ$  Q III +cot



c.  $\frac{4\pi}{7}$  Q II



d.  $-\frac{2\pi}{5}$  Q IV



Just a reminder:

Complimentary angles

Sum up to  $90^\circ$  or  $\frac{\pi}{2}$

Supplementary angles

Sum up to  $180^\circ$  or  $\pi$

Quadrantal Angles:

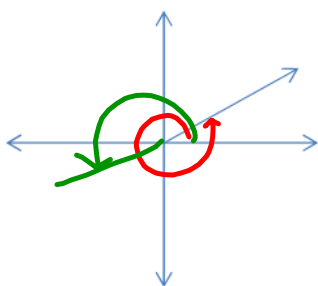
Angles whose terminal side lies along an axis.

**Error Analysis**

Jack was asked to graph a given angle in standard position and to find both a positive and a negative coterminal angle.

The given angle was  $\theta = \frac{7\pi}{6}$ . Determine if Jack graphed the angle correctly and if his answers are correct. If they are not correct provide the correct answer and what he did incorrectly.

Graph



Positive Coterminal Angle:

$$\frac{7\pi}{6} + 2\pi = \frac{7\pi}{6} + \frac{\cancel{12\pi}}{\cancel{6}} = \frac{19\pi}{6}$$

Negative Coterminal Angle:

$$\frac{7\pi}{6} - 2\pi = \frac{7\pi}{6} - \frac{12\pi}{6} = -\frac{5\pi}{6}$$

**Assignment:** Day 1 ws Standard Position & Coterminal Angles - Due 1/20/17

## HPC January 20, 2017

**Learning Objective:** *Students will find values for all six trigonometric functions for angles in any quadrant on a coordinate system.*

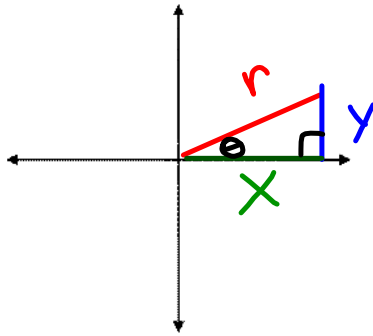
- Check-In Day 1 ws Standard Position and Coterminal Angles
- Complete notes and practice problems for Angle Measures and Trigonometric Functions.

Angle measure and Trigonometric Functions

- Students will find values for all six Trigonometric Functions for angles in any quadrant on a coordinate system

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**Values and signs of the Trigonometric functions in any quadrant:**



$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

What are reciprocal functions?

(cos, sin)  
↓ ↓  
(x, y)

"Standard" Trigonometric Functions

Reciprocal Trigonometric Functions

Sine  $\sin \theta = \frac{y}{r}$

Cosecant  $\csc \theta = \frac{r}{y}$

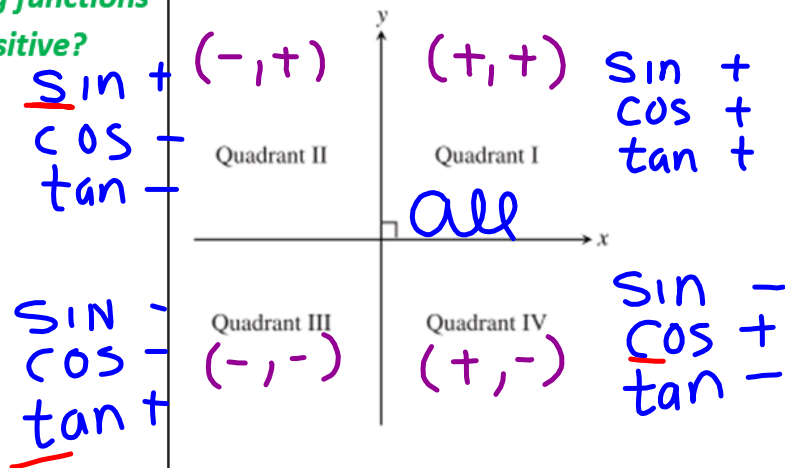
Cosine  $\cos \theta = \frac{x}{r}$

Secant  $\sec \theta = \frac{r}{x}$

Tangent  $\tan \theta = \frac{y}{x}$

Cotangent  $\cot \theta = \frac{x}{y}$

Where are trig functions positive?



Angle Measure and Trigonometric Functions

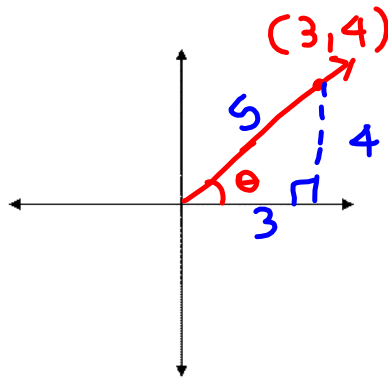
- Students will find values for all six Trigonometric Functions for angles in any quadrant on a coordinate system

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Finding values of Trig functions given an ordered pair

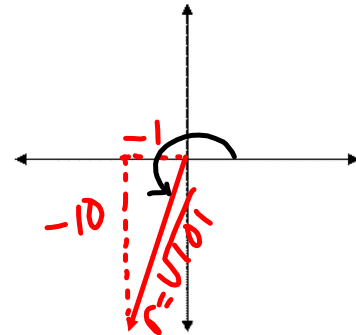
**Example 2:** For the angles whose terminal sides pass through the given point, determine in which quadrant the terminal side of the angle lies, sketch the angle in standard position, and evaluate all six Trigonometric functions of that angle.

a. (3, 4) **Q I**  
 $r = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$



$$\begin{aligned} \sin \theta &= \frac{4}{5} & \csc \theta &= \frac{5}{4} \\ \cos \theta &= \frac{3}{5} & \sec \theta &= \frac{5}{3} \\ \tan \theta &= \frac{4}{3} & \cot \theta &= \frac{3}{4} \end{aligned}$$

b. (-1, -10) **Q III**  
 $r = \sqrt{101}$



$$\begin{aligned} \sin \theta &= \frac{-10}{\sqrt{101}} = -\frac{10\sqrt{101}}{101} & \csc \theta &= -\frac{\sqrt{101}}{10} \\ \cos \theta &= \frac{-1}{\sqrt{101}} = -\frac{\sqrt{101}}{101} & \sec \theta &= -\sqrt{101} \\ \tan \theta &= 10 & \cot \theta &= \frac{1}{10} \end{aligned}$$



Angle Measure and Trigonometric Functions

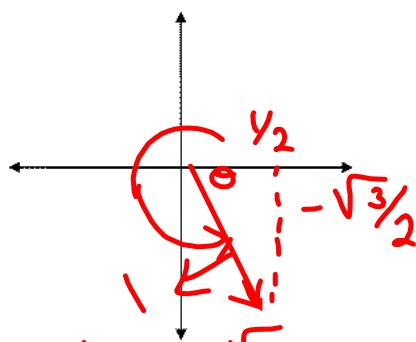
- Students will find values for all six Trigonometric Functions for angles in any quadrant on a coordinate system

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Finding values of Trig functions given an ordered pair

**Example 3:** Find the values for the six Trig functions for the following angles:

a.  $300^\circ$   $(\frac{1}{2}, -\frac{\sqrt{3}}{2})$



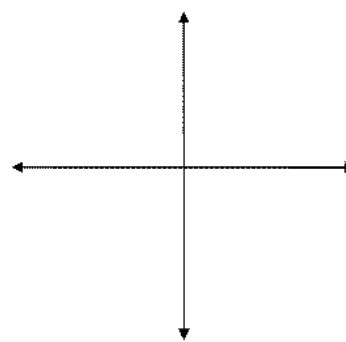
$x = \frac{1}{2}$   $y = -\frac{\sqrt{3}}{2}$   $r = 1$

$\sin \theta = -\frac{\sqrt{3}}{2}$   $\csc \theta = -\frac{2\sqrt{3}}{3}$

$\cos \theta = \frac{1}{2}$   $\sec \theta = 2$

$\tan \theta = -\frac{\sqrt{3}}{1}$   $\cot \theta = -\frac{1}{\sqrt{3}}$

b.  $-\frac{3\pi}{4}$



$x =$   $y =$   $r =$

$\sin \theta =$   $\csc \theta =$

$\cos \theta =$   $\sec \theta =$

$\tan \theta =$   $\cot \theta =$

Angle Measure and Trigonometric Functions

- Students will find values for one trigonometric function when given the value of a second trigonometric function

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Using one Trig function to find others.

Example 4:

$$X = -\sqrt{65} \quad Y = 4 \quad r = 9$$

Q II

If  $\sin \theta = \frac{4}{9}$  and  $\tan \theta < 0$ , find  $\cos \theta$  and  $\tan \theta$ 

$$\sin \theta = \frac{4}{9} = \frac{y}{r}$$

$$x^2 + y^2 = r^2$$

$$x^2 + 16 = 81$$

$$x^2 = 65$$

$$x = -\sqrt{65}$$

$$\cos \theta = \frac{x}{r} = -\frac{\sqrt{65}}{9}$$

$$\tan \theta = \frac{y}{x} = \frac{4}{-\sqrt{65}}$$

$$\tan \theta = \frac{-4\sqrt{65}}{65}$$

Example 5:

$$X = 2\sqrt{10} \quad Y = 3 \quad r = 7$$

Q I

If  $\csc \theta = \frac{7}{3}$  and  $\cos \theta > 0$ , find  $\cos \theta$  and  $\cot \theta$ 

$$\csc \theta = \frac{7}{3} = \frac{r}{y} \quad \cos \theta = \frac{2\sqrt{10}}{7}$$

$$x^2 + y^2 = r^2 \quad \cot \theta = \frac{2\sqrt{10}}{3}$$

$$x^2 + 9 = 49$$

$$x^2 = 40$$

$$x = \sqrt{40} = \sqrt{4} \sqrt{10} = 2\sqrt{10}$$

HW DR 1, 3, 4, 25, 27